FR

1

$g(x) = \int_0^x f(t)\,dt$

$g'(x) = f(x)$

$g''(x) = f'(x)$

ⓐ $g(4) = \int_0^4 f(t)\,dt = 3$

$g'(4) = f(4) = 0$

$g''(4) = f'(4) = -2$

ⓑ Since $g'(x) = f(x) < 0$ on $(-1, 1)$ & $g'(x) = f(x) > 0$ on $(1, 4)$

there is a relative minimum on g at $x = 1$

(g' changed from $-$ to $+$)

ⓒ Since $g(5) = 2$ & g has a period of 5

$\cancel{g(10) = 2 \cdot g(5)}$ $g(10) = \int_0^{10} f(t)\,dt = 2 \cdot \int_0^5 f(t)\,dt$

$= 2 \cdot 2$

$= 4$

A tangent line to g at $x = 108$:

Point $(108, ?)$

$\hookrightarrow g(108) = \int_0^{108} f(t)\,dt = \underbrace{21 \cdot \int_0^5 f(t)\,dt}_{\text{This would cover from 0 to 105}} + \int_0^3 f(t)\,dt$

$= 21 \cdot 2 + 2$

$= 44$

$\therefore$ Point $(108, 44)$

Slope $= g'(108) = g'(3) = f(3) = 2$

$$\boxed{y - 44 = 2(x - 108)}$$

② $g(x) = \int_{-4}^{x} f(t)\,dt$

$g'(x) = f(x)$

$g''(x) = f'(x)$

ⓐ $g(-1) = \int_{-4}^{-1} f(t)\,dt = 15/2$

$g'(-1) = f(-1) = -2$

$g''(-1) = \underbrace{f'(-1)}_{\text{"Pointy Place"}}$ DNE

ⓑ Point of Inflection occurs when $g''(x) = f'(x)$ changes signs

this occurs when $x = 1$, ∴ there is a point of inflection on g when $x = 1$.

...OR... you could say...

Points of inflection occur when $g'(x) = f(x)$ changes from INC to DEC

or DEC to INC... ~~this~~

$g'(x) = f(x)$ changes from INC to DEC at $x = 1$

ⓒ $h(x) = \int_{x}^{3} f(t)\,dt$

$h(x) = 0$ when $x = 3$: $\int_{3}^{3} f(t)\,dt = 0$

Also when $x = 1$: $\int_{1}^{3} f(t)\,dt = +1 - 1 = 0$

Also when $x = -1$: $\int_{-1}^{3} f(t)\,dt = -1 + 1 + 1 - 1 = 0$

ⓓ h is decreasing when $h'(x) < 0$

$h'(x) = -f(x)$ ∴ when $-f(x) < 0$

$f(x) > 0$, then h is decreasing.

this occurs on $(0, 2)$

3 $g(x) = \int_1^x f(t)\,dt$

$g'(x) = f(x)$

$g''(x) = f'(x)$

a $g(4) = \int_1^4 f(t)\,dt = \frac{5}{2} + \frac{1}{2} - \frac{1}{2} = \boxed{\frac{5}{2}}$

$g(-2) = \int_1^{-2} f(t)\,dt = -\int_{-2}^{1} f(t)\,dt = -8$

b $g'(1) = f(1) = 4$

c Absolute minimums ... Check critical points
Check endpoints

Critical points ... $\underbrace{g'(x) = f(x) = 0}_{\text{at } x=3 \text{ or } x=-2}$ or $\underbrace{\text{undefined}}_{f(x) \text{ always defined on } [-2,4]}$

candidates	$g(x)$
$x = -2$	-8
$x = 3$	3
$x = 4$	$5/2$

$g(3) = \int_1^3 f(t)\,dt = \frac{5}{2} + \frac{1}{2} = 3$

$\therefore$ The minimum occurs when $x = -2$ & the $\boxed{\text{Absolute minimum} = -8}$

d Points of Inflection occur when $g''(x) = f'(x)$ changes signs (or equivalently... when $g'(x) = f(x)$ changes from INC to DEC or DEC to INC)

This occurs only at $x = 1$

(4) $g(x) = \int_{-3}^{x} f(t)\,dt$

$g'(x) = f(x)$

$g''(x) = f'(x)$

(a) $g(0) = \int_{-3}^{0} f(t)\,dt = \frac{9}{2}$

$g'(0) = f(0) = 1$

(b) A relative maximum occurs when $g'(x) = f(x)$ changes signs from + to −.

Since $g'(x) = f(x) > 0$ on $(1, 3)$ &

$g'(x) = f(x) < 0$ on $(3, 4)$

there is a rel. max when $x = 3$.

(c) Abs. min ... check critical points ... $\underbrace{g'(x) = f(x) = 0}_{\text{when } x = -4,\ 1, \text{ and } 3}$ or undefined

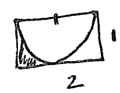
check endpoints ... $[-5, 4]$

candidates	$g(x)$
-5	0
-4	-1
1	$\frac{11}{2} - \frac{\pi}{4} > 0$
3	$\frac{13}{2} - \pi/2 > 0$
4	$6 - \pi/2 > 0$

$g(-5) = \int_{-3}^{-5} f(t)\,dt = 0$

$g(-4) = \int_{-3}^{-4} f(t)\,dt = -\int_{-4}^{-3} f(t)\,dt = -1$

$g(1) = \int_{-3}^{1} f(t)\,dt = \frac{9}{2} + 1 - \frac{\pi}{4}$



$\frac{1}{2}\left(2 \cdot 1 - \frac{1}{2}\pi(1)^2\right)$

$\frac{1}{2}\left(2 - \frac{\pi}{2}\right)$

$\boxed{\therefore \text{ the Absolute minimum} = -1}$

(occurs when $x = -4$).

(d) Point of Inflection occurs when $g''(x) = f'(x)$ changes signs)

(or equivalently ... when $g'(x) = f(x)$ changes from INC to DEC or DEC to INC)

This occurs at $x = -3$, $x = 1$, and $x = 2$.

$g(3) = \int_{-3}^{3} f(t)\,dt = \frac{9}{2} + \left(2 - \frac{\pi}{2}\right)$

$g(4) = \int_{-3}^{4} f(t)\,dt = \frac{9}{2} + \left(2 - \frac{\pi}{2}\right) - \frac{1}{2}$

5

(a) $R'(45) \approx \dfrac{R(50) - R(40)}{50 - 40} = \dfrac{55 - 40}{10} = \dfrac{15}{10} = 1.5 \text{ gal/min}^2$

or

$\dfrac{\text{gal/min}}{\text{min}}$

(b) Since there is a max of $R'(t)$ at $t = 45$... $R''(45) = 0$

(c) $\int_0^{90} R(t)\,dt \approx 30(20) + 10(30) + 10(40) + 20(55) + 20(65) = 3700$

Since $R(t)$ is Increasing a Left Riemann Sum will be less than $\int_0^{90} R(t)\,dt$

(d) $\int_0^b R(t)\,dt$ = Total # of gallons of fuel in b minutes consumed by the plane

$\frac{1}{b}\int_0^b R(t)\,dt$ = Avg # of gal/min in b minutes

⑥ ⓐ $W'(12) \approx \dfrac{W(15)-W(9)}{15-9} = \dfrac{21-24}{6} = -\frac{1}{2}\ {}^{\circ}C/\text{day}$

Or

$\dfrac{W(12)-W(9)}{12-9} = \dfrac{22-24}{3} = -\frac{2}{3}\ {}^{\circ}C/\text{day}$

OR

$\dfrac{W(15)-W(12)}{15-12} = \dfrac{21-22}{3} = -\frac{1}{3}\ {}^{\circ}C/\text{day}$

$\Delta t = 3$

ⓑ $\text{Avg Temp} = \dfrac{\int_0^{15} W(t)\,dt}{15-0} = \dfrac{3\left(\frac{20+31}{2}\right) + 3\left(\frac{31+28}{2}\right) + 3\left(\frac{28+24}{2}\right) + 3\left(\frac{24+22}{2}\right) + 3\left(\frac{22+21}{2}\right)}{15}$

$= 25.1\ {}^{\circ}C$

ⓒ $P(t) = 20 + 10te^{(-t/3)}$

$\underbrace{P'(12) \approx -.549}_{\text{used calculator...}}$ Temperature is decreasing .549°C/day on day 12

ⓓ Avg value of $P(t) = \underbrace{\dfrac{\int_0^{15} P(t)\,dt}{15-0}}_{\text{used calculator...}} = 25.757\ {}^{\circ}C$

⑦ $R(t)$ is mosquitoes/day ... ALREADY a RATE of change in mosquitoes!

(a) since $R(6) \approx 4.438 > 0$, the # of mosquitoes is increasing.

(b) $R'(6) = -1.913 < 0$ ... ∴ Since $\left.\begin{array}{l} R'(6) < 0 \\ \& \\ R(6) > 0 \end{array}\right\}$ the # of mosquitoes is increasing at a decreasing rate

(c) TOTAL # OF MOSQUITOES = # originally + Change in # of mosquitoes

$$= 1000 + \int_0^{31} R(t)\,dt$$

$$= 1000 - 36$$

$$\boxed{= 964}$$

(d) ABS MAX ... occurs when $R(t) = 0$ or @ endpoints

already the Rate!

$R(t) = 0$ when $t = 7.8539816$ or 23.561945

$$\boxed{\text{total \# of mosquitoes at any time } x} = 1000 + \int_0^{x} R(t)\,dt$$

Candidates	# of mosquitoes	
$x = 0$	1000	(given)
$x = 7.8539816$	1039	$\approx 1000 + \int_0^{7.8539816} R(t)\,dt$
$x = 23.561945$	842	$\approx 1000 + \int_0^{23.561945} R(t)\,dt$
$x = 31$	964	(from part c)

$\boxed{\therefore \text{ The MAX \# of mosquitoes is } 1039.}$

(more of a preview of what's to come ...)

MC

① Point $(2, 10)$

$f(2) = 3(2)^2 - 2(2) + 2$

Slope $= f'(2) = 6(2) - 2 = 10$

$f'(x) = 6x - 2$

$y - 10 = 10(x - 2)$

$y = 10x - 20 + 10$

$y = 10x - 10$

$\boxed{D}$

② Point $(3, \frac{10}{3})$

$f(3) = 3 + \frac{1}{3} = \frac{10}{3}$

Slope $= f'(3) = 1 - \frac{1}{3^2} = \frac{8}{9}$

$f'(x) = 1 - \frac{1}{x^2}$

$y - \frac{10}{3} = \frac{8}{9}(x - 3)$

$y = \frac{8}{9}x - \frac{8}{3} + \frac{10}{3}$

$y = \frac{8}{9}x + \frac{2}{3}$

$\boxed{C}$

③ $y = \sqrt[3]{x} = x^{1/3}$

Point $(8, 2)$

$\sqrt[3]{8}$

Slope $= y'(8) = \frac{1}{3}(8)^{-2/3} = \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$

$y' = \frac{1}{3}x^{-2/3}$

$y - 2 = \frac{1}{12}(x - 8)$

Plug in $x = 13$

$y = \frac{1}{12}(13 - 8) + 2$

$y = \frac{1}{12}(5) + 2$

$y \approx 2\frac{5}{12} \approx 2.4167$

$\boxed{A}$

④ $V = \frac{4}{3}\pi r^3$

$dV = \frac{4}{\cancel{3}}\pi[\cancel{3}r^2 dr]$

$dV = 4\pi r^2 dr$

when $r = 1$ & $dr = .1$ ... $dV = 4\pi(1)^2(.1) \approx 1.3$

$\boxed{A}$

⑤ $V_{sphere} = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 90\pi$ ft³/min ... Find dr/dt when $r=3$

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

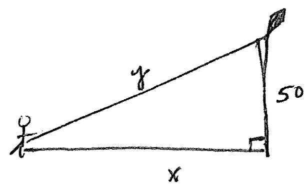
$$90\cancel{\pi} = 4\cancel{\pi}(3)^2 \cdot dr/dt$$

$$\frac{90}{4 \cdot 3^2} = \frac{dr}{dt}$$

$$\frac{5}{2} = \frac{dr}{dt}$$

$\boxed{C}$

⑥



$$x^2 + 50^2 = y^2$$

$$2x\, dx/dt = 2y\, dy/dt$$

$dx/dt = 10$ m/sec ...

Find dy/dt when $y=130$

when $y=130$

$x^2 + 50^2 = 130^2$

$x = 120$

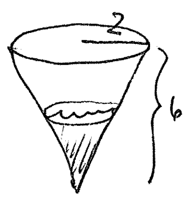
$$2(120)(10) = 2(130)\, dy/dt$$

$$\frac{2(120)(10)}{2(130)} = dy/dt$$

$$9.2 \approx dy/dt$$

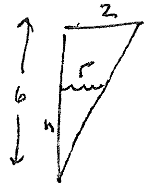
$\boxed{B}$

⑦



$V = \frac{1}{3}\pi r^2 h$

$dh/dt = -.9$ in/sec



$\frac{r}{h} = \frac{2}{6}$

$r = \frac{1}{3}h$

Find dV/dt when $h=3$

$$V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{1}{9}h^2\right)h$$

$$\boxed{V = \frac{\pi}{27}h^3}$$

$$\frac{dV}{dt} = \frac{\pi}{27} \cdot 3h^2\, dh/dt$$

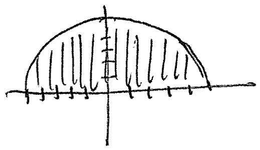
$$= \frac{\pi}{27} \cdot 3(3)^2(-.9)$$

$$\approx -2.83 \text{ in}^3/\text{sec}$$

$\boxed{C}$

(8) $\displaystyle\int_{-5}^{5} \underbrace{\sqrt{25-x^2}}\,dx = \frac{1}{2}\pi(5)^2 = \boxed{\frac{25\pi}{2}}$ $\boxed{B}$

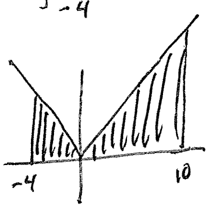
top ½ of circle ... $R=5$ $c(0,0)$



(9) $\displaystyle\int_{-4}^{10} |x|\,dx = \frac{1}{2}(4)(4) + \frac{1}{2}(10)(10)$

$= 8 + 50$ $\boxed{B}$

$= 58$



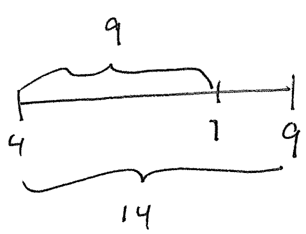
(10) $\displaystyle\int_7^{11} 5f(x)\,dx + \int_7^{11} g(x)\,dx = 5\underbrace{\int_7^{11} f(x)\,dx} + \underbrace{\int_7^{11} g(x)\,dx}$

$= 5\ (-2) + 9$ $\boxed{B}$

$= -10 + 9$

$= -1$

(11)



$\displaystyle\int_4^7 g(x)\,dx + \int_7^9 g(x)\,dx = \int_4^9 g(x)\,dx$

$\displaystyle 9 + \int_7^9 g(x)\,dx = 14$

$\displaystyle\int_7^9 g(x)\,dx = 5$ $\boxed{B}$

$\displaystyle\Rightarrow \int_9^7 g(x)\,dx = \boxed{-5}$

(12) $\frac{d}{dx}\left[\int_0^x \sqrt{8t+3}\,dt\right] = \sqrt{8x+3}$ $\quad$ (D)

(13) $\frac{d}{dx}\left[\int_0^{\sin x} \frac{1}{9-t^2}\,dt\right] = \frac{1}{9-(\sin x)^2} \cdot \cos x = \frac{\cos x}{9-\sin^2 x}$ $\quad$ $\boxed{D}$

(14) $\frac{d}{dx}\left[\int_0^{\sqrt{x}} 5t\cos(t^{10})\,dt\right] = 5\sqrt{x}\,\cos\left((\sqrt{x})^{10}\right) \cdot \frac{1}{2}x^{-1/2}$

$= 5\cancel{\sqrt{x}}\cos(x^5) \cdot \frac{1}{2\cancel{\sqrt{x}}}$ $\quad$ $\boxed{A}$

$= \frac{5}{2}\cos(x^5)$

(15) $\int_3^{-1} 4^x\,dx = \left.\frac{4^x}{\ln 4}\right|_3^{-1} = \frac{4^{-1}}{\ln 4} - \frac{4^3}{\ln 4}$

$= \frac{1}{4\ln 4} - \frac{64}{\ln 4}$ $\quad$ $\boxed{B}$

$= \frac{1}{4\ln 4} - \frac{(64)(4)}{4\ln 4}$

$= \frac{-255}{4\ln 4}$

(16) $\int_{1/4}^{3}\left(4-\frac{1}{x}\right)dx = \left.4x - \ln|x|\right|_{1/4}^{3} = \left[4(3) - \ln(3)\right] - \left[4(1/4) - \ln(1/4)\right]$

$= 12 - \ln 3 - 1 + \ln 1/4$

$= 11 - \ln 3 + \ln 4^{-1}$ $\quad$ $\boxed{A}$

$= 11 - \ln 3 - \ln 4$

$= 11 - (\ln 3 + \ln 4)$

$= 11 - \ln(12)$

(17) $$\int_{-\pi/4}^{3\pi/4} 8 \sec\theta \tan\theta \, d\theta = 8 \int_{-\pi/4}^{3\pi/4} \sec\theta \tan\theta \, d\theta$$

$$= 8\left(\sec\theta \Big|_{-\pi/4}^{3\pi/4}\right)$$

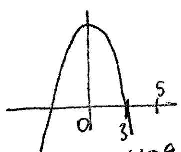
$$= 8\left[\sec\left(\tfrac{3\pi}{4}\right) - \sec\left(-\pi/4\right)\right]$$

(C)

$$= 8\left[-\sqrt{2} - \sqrt{2}\right]$$

$$= -16\sqrt{2}$$

(18)



$y = 9 - x^2$

TOTAL AREA: $\int_0^5 |9-x^2| \, dx$ ... or w/o a calculator ...

$$= \int_0^3 (9-x^2)\,dx - \int_3^5 (9-x^2)\,dx$$

$$= \left[9x - \frac{x^3}{3}\Big|_0^3\right] - \left[9x - \frac{x^3}{3}\Big|_3^5\right]$$

$$= \left(\left[9(3) - \frac{3^3}{3}\right] - \left[9(0) - \frac{0^3}{3}\right]\right) - \left(\left[9(5) - \frac{5^3}{3}\right] - \left[9(3) - \frac{3^3}{3}\right]\right)$$

$$= (27 - 9 - 0) - \left(45 - \frac{125}{3} - 27 + 9\right)$$

$$= (18) - \left(27 - \frac{125}{3}\right)$$

$$= -9 + \frac{125}{3}$$

(B)

$$= \frac{-27 + 125}{3}$$

$$= \frac{98}{3}$$

(19) if $\int_1^x f(t)\,dt = 5x^2 + 7x - 3$

AND

$\frac{d}{dx}\left(\int_1^x f(t)\,dt\right) = \underline{f(x)}$ then $f(x) = 10x + 7$

Derivative of $5x^2 + 7x - 3$

(B)

(20) Linearization: $\underline{\text{Point } (0, 9)}$

$f(0) = 9 + \int_1^{0+1} \tan\left(\frac{\pi t}{4}\right) dt = 9 + \underbrace{\int_1^1 \tan\left(\frac{\pi t}{4}\right) dt}$

$= 9 + 0$

$= 9$

$\underline{\text{Slope}} = f'(0) = \tan\left(\frac{\pi(0+1)}{4}\right) = \tan\left(\frac{\pi}{4}\right) = \underline{1}$

$f'(x) = \tan\left(\frac{\pi(x+1)}{4}\right) \cdot 1$

(D)

$y - 9 = 1(x - 0)$

$\boxed{y = x + 9}$