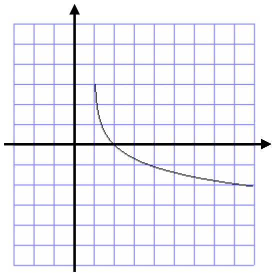
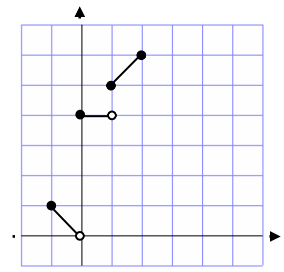
Calculus
Chapter 1 In Class Review

**For questions 1 and 2, sketch each without a calculator.**

1. $y = -\log(x-1)$
Reflect across the $x$ axis … Right 1
Asymptote at $x = 1$.



2. $y = \begin{cases} -x & -1 \le x < 0 \\ 4 & 0 \le x < 1 \\ x+5 & 1 \le x \le 2 \end{cases}$



3. Prove whether the function is odd or even: $y = x^3 - 5x$

To prove, plug in "–$x$" to the function … $f(-x) = (-x)^3 - 5(-x) = -x^3 + 5x = -(x^3 - 5x) = -f(x)$

$\therefore f(x)$ is ODD

4. Change to base 10. $\log_7 14$

Use the change of base formula … $\dfrac{\log 14}{\log 7}$

5. Find $k$ in the equation $3y + kx = 4$
   a) to make the line horizontal
horizontal line has a slope of zero … $y$ values are always the same … equation of the form $y = \#$ … thus, $k = 0$.

   b) to make the line parallel to $y = 3x + 5$

parallel lines have the same slope … slope of this line is 3 … slope of original line would be $-\dfrac{k}{3}$ … setting these equal to each other and solving gives you $k = -9$.

6. Find the equation of the line perpendicular to $y = -3x + 5$ that goes through (4, 1).
Perpendicular lines have slopes that are opposites and reciprocals … that would mean your new line would have slope = $\frac{1}{3}$.
Using point – slope form … $y - 1 = \frac{1}{3}(x-4)$.

7. Change to a Cartesian equation: $x(t) = 2\sec(t)$
$y(t) = \tan(t) - 1$

Use the trig identity … $1 + \tan^2 x = \sec^2 x$. Solve the equations above for tan $x$ and sec $x$ and substitute this into the identity to get

$$1 + (y+1)^2 = \left(\frac{x}{2}\right)^2$$

This becomes the standard equation for a hyperbola if you rewrite it like

$$1 = \frac{x^2}{4} - (y+1)^2$$

8. Solve for $x$ given the domain restrictions.

a) $\sin^{-1}\left(\frac{1}{2}\right)=x$ b) $\sin x=\frac{1}{2}$ if $0\le x\le 2\pi$ c) $\sin x=\frac{1}{2}$ if $-\infty<x<\infty$

a) $x=\dfrac{\pi}{6}$

b) $x=\dfrac{\pi}{6}$ and $\dfrac{5\pi}{6}$

c) $x=\dfrac{\pi}{6}+k\cdot 2\pi$ where $k$ is an integer.

$x=\dfrac{5\pi}{6}+k\cdot 2\pi$

9. $k(x)$ is shown.

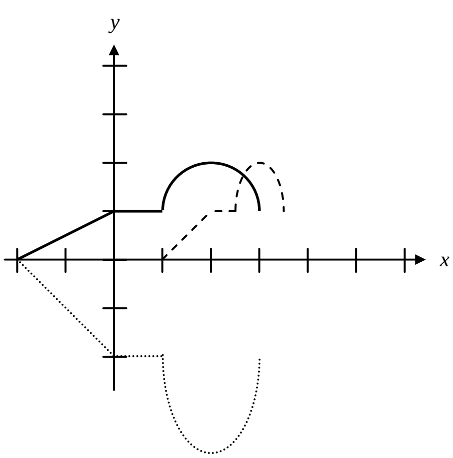
a) Graph $y=-2k(x)$ (dotted)

Reflect over $x$-axis … vertical stretch (2)

b) Graph $y=k(2x-4)$ (dashed)

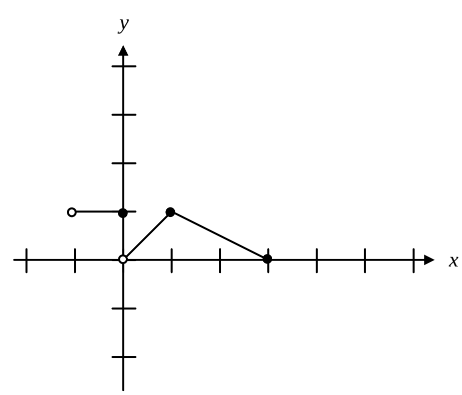
Rewrite $y=k(2(x-2))$

Horizontal shrink (1/2) … right 2




10. $h(x)$ is shown. Write the equation for $h(x)$.

$$h(x)=\begin{cases}1 & -1<x\le 0\\ x & 0<x\le 1\\ -\frac{1}{2}x+\frac{3}{2} & 1<x\le 3\end{cases}$$




11. If $\ln(x)-\ln\left(\frac{1}{x}\right)=2$, solve for $x$.

$$\ln x-\ln\left(\frac{1}{x}\right)=2$$
$$\ln\left(\frac{x}{1/x}\right)=2$$
$$\ln\left(x^2\right)=2$$
$$e^2=x^2$$
$$e=x$$

NOTE: $x = -e$ also according to the algebra, BUT this value is NOT in the domain of ln.

12. If $f(x)=\dfrac{4}{x-1}$ and $g(x)=2x$, then the solution set of $f(g(x))=g(f(x))$ is

First, $f(g(x))=f(2x)=\dfrac{4}{2x-1}$, and then $g(f(x))=g\left(\dfrac{4}{x-1}\right)=2\left(\dfrac{4}{x-1}\right)=\dfrac{8}{x-1}$

Setting them equal to each other and solving gives you

$$\begin{aligned} f(g(x)) &= g(f(x)) \\ \frac{4}{2x-1} &= \frac{8}{x-1} \\ 4x-4 &= 16x-8 \\ 4 &= 12x \\ \frac{1}{3} &= x \end{aligned}$$

A) $\frac{1}{3}$ B) 2 C) 3 D) -1 and 2 E) $\frac{1}{3}$ and 2

13. $\ln(x-2)<0$ if and only if

The ln function is only negative between 0 and 1 … for this function there is a translation of 2 to the right. So, this function is only negative between 2 and 3.

A) $x < 3$ B) $0 < x < 3$ C) $2 < x < 3$ D) $x > 2$ E) $x > 3$

14. Which of the following define a function *f* for which $f(-x)=-f(x)$?

This is the definition of an odd function … instead of trying all 5 of these, just pick the one that is odd.

A) $f(x)=x^2$ B) $f(x)=\sin x$ C) $f(x)=\cos x$ D) $f(x)=\log x$ E) $f(x)=e^x$