AP Calculus
Chapter 5 Review

Name: KEY
Block: ____

1. [No Calculator] Evaluate using the FTOC (the evaluation part)

a) $\int_2^7 \left(\frac{8}{x}+7\sqrt[3]{x}+x^{-4}\right) dx = \int_2^7 \left(8\cdot\frac{1}{x}+7x^{1/3}+x^{-4}\right)dx$

$= 8\ln|x| + 7\cdot\frac{3}{4}x^{4/3} - \frac{1}{3}x^{-3}\Big|_2^7$

$\boxed{= \left[8\ln|7| + \frac{21}{4}(7)^{4/3} - \frac{1}{3}(7)^{-3}\right] - \left[8\ln|2| + \frac{21}{4}(2)^{4/3} - \frac{1}{3}(2)^{-3}\right]}$

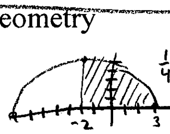
b) $\int_4^9 \left(\frac{5}{x^3}+7\sqrt{x}+\frac{1}{x}\right) dx = \int_4^9 \left(5x^{-3}+7x^{1/2}+\frac{1}{x}\right)dx$

$= 5\cdot\left(-\frac{1}{2}\right)x^{-2} + 7\cdot\frac{2}{3}x^{3/2} + \ln|x|\Big|_4^9$

$\boxed{= \left[-\frac{5}{2}(9)^{-2} + \frac{14}{3}(9)^{3/2} + \ln|9|\right] - \left[-5/2\,(4)^{-2} + \frac{14}{3}(4)^{3/2} + \ln|4|\right]}$

2. [No Calculator] Evaluate using geometry

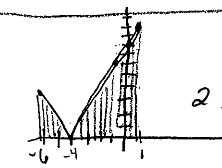
a) $\int_{-2}^{3} \sqrt{25-(x+2)^2}\, dx$



$\frac{1}{4}$ of a circle with radius = 5

$\frac{1}{4}\pi(5)^2 = \boxed{\frac{25\pi}{4}}$

c) $\int_{-6}^{1} |8+2x|\, dx$



$|2(x+4)|$

2 Δs...

$= \frac{1}{2}(2)(4) + \frac{1}{2}(5)(10)$

$= 4 + 25$

$= \boxed{29}$

3. [No Calculator] Evaluate each derivative.

a) $\frac{d}{dx}\left[\int_{10}^{x} \tan(3t^2+9)\, dt\right] = \boxed{\tan(3x^2+9)}$

b) Find $h'(x)$ if $h(x) = \int_{5x^4}^{\sec x} \sqrt{4t-9}\, dt$.

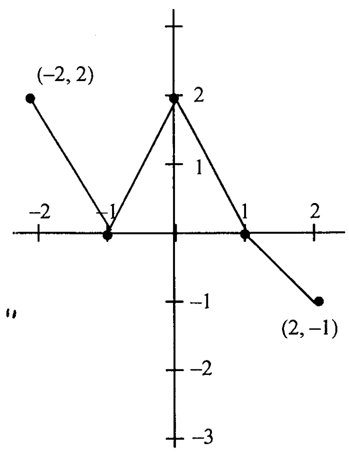
$\boxed{h'(x) = \sqrt{4\sec x - 9}\cdot(\sec x \tan x) - \sqrt{4(5x^4)-9}\cdot(20x^3)}$

c) $\frac{d}{dx}\left[\int_{8}^{x} \ln(3t^2+9)\, dt\right] = \boxed{\ln(3x^2+9)}$

d) Find $h'(x)$ if $h(x) = \int_{3x^6}^{\tan x} \frac{9}{x^2-1}\, dt$.

$\boxed{h'(x) = \frac{9}{\tan^2 x - 1}\cdot(\sec^2 x) - \frac{9}{(3x^6)^2-1}\cdot(18x^5)}$

4. [No Calculator] Given the graph of $f(x)$ as shown and the definition of $g(x) = \int_0^x f(t)\, dt$



Graph of f

a) Find $g(-1)$, $g'(-1)$, $g''(-1)$

$g(-1) = \int_0^{-1} f(t)\,dt = -\underbrace{\int_{-1}^{0} f(t)\,dt}_{\frac{1}{2}(1)(2)=1} = -1$

$g'(x) = f(x)$

$\therefore g'(-1) = f(-1) = 0$

$g''(x) = f'(x)$

$\therefore g''(-1) = f'(-1)$ $\boxed{\text{DNE}}$ because of a "pointy place"

b) Over what interval is $g(x)$ increasing. Show your work and explain your reasoning.

$g(x)$ is increasing if $g'(x) > 0$

Since $g'(x) = f(x)$, $g(x)$ is increasing on $[-2, 1]$.

c) Over what interval is $g(x)$ concave up? Show your work and explain your reasoning.

$g(x)$ is concave up if $g''(x) > 0$

Since $g''(x) = f'(x)$, $g(x)$ is concave up on $(-1, 0)$

d) Graph $g(x)$

First find some points...

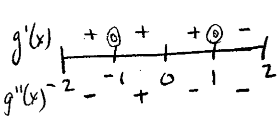
$g(-2) = \int_0^{-2} f(t)\,dt = -\underbrace{\int_{-2}^{0} f(t)\,dt}_{\frac{1}{2}(1)(2)+\frac{1}{2}(1)(2)} = -2$

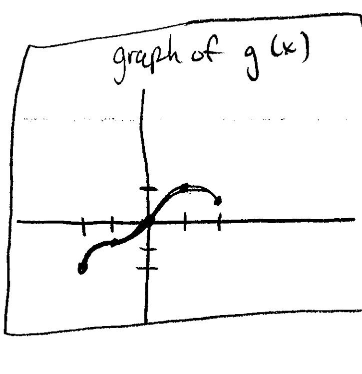
Already know $\underbrace{g(-1) = -1}_{\text{part (a)}}$

$g(0) = \int_0^0 f(t)\,dt = 0$

$g(1) = \int_0^1 f(t)\,dt = 1$

$g(2) = \int_0^2 f(t)\,dt = \frac{1}{2}$

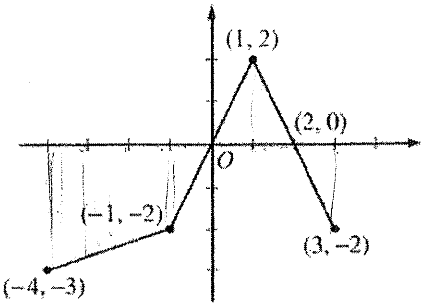




5. [No Calculator] The graph of the function f shown below consists of three line segments.

a) Let g be the function given by $g(x)=\int_{-4}^{x} f(t)\,dt$.

For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.



Graph of f

$g(-1)=\int_{-4}^{-1} f(t)dt = \boxed{\frac{-15}{2}}$

Trapezoid

$3\left(\frac{-2+-3}{2}\right)$

$g'(x)=f(x)$

$\boxed{\therefore g'(-1)=f(-1)=-2}$

$g''(x)=f'(x)$

$\therefore g''(-1)=f'(-1)$

DNE ... "pointy place"

b) For the function g defined in part a, find the x-coordinate of each point of inflection of the graph of g on the open interval $-4<x<3$. Explain your reasoning.

$g(x)$ has a point of inflection when $g''(x)$ changes signs. Since $g''(x)=f'(x)$ we need to look for x-coordinates where $f'(x)$ changes signs... this occurs at $\boxed{x=1}$ only

c) Let h be the function given by $h(x)=\int_{x}^{3} f(t)\,dt$.

Find all values of x in the closed interval $-4\le x\le 3$ for which $h(x)=0$.

$h(x)=0$ when $\boxed{x=3}$ since $\int_3^3 f(t)dt=0$ and

$h(x)=0$ when $\boxed{x=1}$ since $h(1)=\int_1^3 f(t)dt = 1-1=0$ and

$h(x)=0$ when $x=-1$ since $h(-1)=\int_{-1}^{3} f(t)\,dt = -1+1+1-1=0$

d) For the function h defined in part c, find all intervals on which h is decreasing. Explain your reasoning.

h is decreasing if $h'(x)<0$

Since $h'(x)=-f(x)$, $h'(x)<0$ when $f(x)>0$... this occurs on $\boxed{[0,2]}$

6. [Calculator] The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table below shows the water temperature as recorded every 3 days over a 15-day period.

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

$W'(12)$ is the slope of W at $t=12$ $\quad W'(12)\approx \frac{21-24}{15-9} = \frac{-3}{6} = -\frac{1}{2}$ °C/day

the temp of the water is decreasing approximately 1/2 °C/day at day 12.

b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0\le t\le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t=3$ days.

Average temp = Average Value of $W(t)$

$\frac{\int_0^{15} W(t)dt}{15} = \frac{3\left(\frac{31+20}{2}\right)+3\left(\frac{28+31}{2}\right)+3\left(\frac{24+28}{2}\right)+3\left(\frac{22+24}{2}\right)+3\left(\frac{21+22}{2}\right)}{15} = \frac{376.5}{15} = \boxed{25.1\text{ °C}}$

c) A student proposes the function P, given by $P(t)=20+10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

$P'(12)\approx -.549$

The temperature of the water is Decreasing .549 °C/day at the time $t=12$

d) Use the function P defined in part c to find the average value, in °C, of $P(t)$ over the time interval $0\le t\le 15$ days.

$\frac{\int_0^{15} P(t)\,dt}{15} \approx \boxed{25.757\text{ °C}}$

7. [Calculator] For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

a) Show that the number of mosquitoes is increasing at time $t = 6$.

$R(x)$ is already a Rate of change! So $R(x)$ is treated like a "derivative function"

$R(6) \approx 4.438 > 0$

$\boxed{\text{Since } R(6) > 0, \text{ the \# of mosquitoes is increasing at } t=6}$

b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

We just showed $R(6) > 0$ to show the # of mosquitoes is increasing,

Now, since $R'(6) \approx -1.913 < 0$ the # of mosquitoes is $\underbrace{\text{increasing}}_{R(6)>0}$ at a $\underbrace{\text{decreasing rate}}_{R'(6)<0}$

c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

TOTAL # of mosquitoes = starting # + change in mosquitoes

$= 1000 + \int_0^{31} R(t)\,dt \approx 964.335192$ $\boxed{964}$

d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$? Show the analysis that leads to your conclusion.

$T(x)$ = total # of mosquitoes at time x days

$\boxed{T(x) = 1000 + \int_0^x R(t)\,dt}$ ← MAXIMIZE THIS! on $[0, 31]$

$T'(x) = R(x) = 0$ when $x \approx 0$, $\underbrace{x \approx 7.8539816}_{\text{STORE AS A}}$, $\underbrace{x \approx 23.561945}_{\text{STORE AS B}}$

use a candidates TEST...

Rounded

x	0	A	B	31
$T(x)$	1000	1039	842	964

$\boxed{\therefore \text{ the maximum \# of mosquitoes is } \approx 1039 \text{ after } \approx 7.854 \text{ days}}$

8. [No Calculator] Suppose $\int_1^2 f(x)\,dx = 3$, $\int_1^5 f(x)\,dx = -13$, and $\int_1^5 g(x)\,dx = 7$. Find each of the following:

a) $\int_3^3 g(x)\,dx = \boxed{0}$

b) $\int_5^1 f(x)\,dx = -\int_1^5 f(x)\,dx$

$= -(-13)$

$= \boxed{13}$

c) $\int_1^5 [g(x) - f(x)]\,dx$

$= \int_1^5 g(x)\,dx - \int_1^5 f(x)\,dx$

$= 7 - (-13) = \boxed{20}$

d) $\int_2^5 f(x) = \int_1^5 f(x)\,dx - \int_1^2 f(x)\,dx$

$= -13 - (3)$

$= \boxed{-16}$

e) $\int_1^5 [3f(x) - g(x)]\,dx$

$3\int_1^5 f(x)\,dx - \int_1^5 g(x)\,dx$

$3(-13) - (7)$

$-39 - 7$

$\boxed{-46}$

f) $\int_1^5 \frac{g(x)}{4}\,dx = \frac{1}{4}\int_1^5 g(x)\,dx$

$= \frac{1}{4}(7)$

$= \boxed{\frac{7}{4}}$

9. [No Calculator] Suppose $H(x)=\int_2^x \ln(t+5)\,dt$ for the interval [2, 10].

a) Use MRAM to approximate $H(10)$ using 4 equal subdivisions.

$2 \; 4 \; 6 \; 8 \; 10$

$3 \; 5 \; 7 \; 9$

$H(10) = \int_2^{10} \ln(t+5) \approx \boxed{2\cdot\ln(3+5) + 2\cdot\ln(5+5) + 2\cdot\ln(7+5) + 2\cdot\ln(9+5)}$

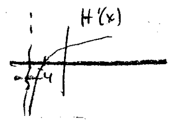
$= 2\cdot\ln(8) + 2\cdot\ln(10) + 2\cdot\ln(12) + 2\cdot\ln(14)$

$= 2\cdot\ln(13440)$

b) When is $H(x)$ decreasing? Justify your response.

if $H'(x) < 0$, then $H(x)$ is decreasing.

$H'(x) = \ln(x+5)$



... $\therefore$ $H(x)$ is decreasing on $(-5,-4)$

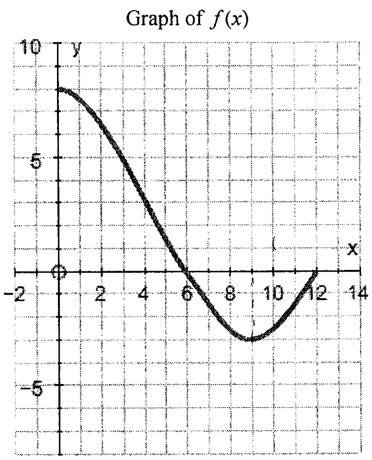
c) If the average rate of change of $H(x)$ on [2, 10] is k, what is the value of $\int_2^{10} \ln(t+5)\,dt$ in terms of k.

slope of $H(x)$ on $[2,10]$

$\boxed{k = \frac{H(10)-H(2)}{10-2}} = \frac{\int_2^{10}\ln(t+5) - \int_2^2 \ln(t+5)}{8} = \frac{\int_2^{10}\ln(t+5)\,dt}{8}$ $\quad \boxed{\therefore \int_2^{10} \ln(t+5) = 8k}$

10. [No Calculator] Let $H(x)=\int_0^x f(t)\,dt$, where f is the continuous function with domain [0, 12] shown below.

Graph of $f(x)$



a) Find $H(0)$

$H(0) = \int_0^0 f(t)\,dt = \boxed{0}$

b) Is $H(12)$ positive or negative? Explain.

$H(12) = \int_0^{12} f(t)\,dt > 0$ because there is more area above the x-axis from 0 to 6 than below the x-axis from 6 to 12

c) Find $H'(x)$ and use it to evaluate $H'(0)$.

$\boxed{H'(x) = f(x)}$ $\quad \boxed{\therefore H'(0) = f(0) = 8}$

d) When is $H(x)$ increasing? Justify your answer.

if $H'(x) > 0$, then $H(x)$ is increasing. Since $H'(x) = f(x)$

$\boxed{H(x) \text{ is increasing on } [0,6]}$

e) Find $H''(x)$.

$\boxed{H''(x) = f'(x)}$

f) When is $H(x)$ concave up? Justify your answer.

$H(x)$ is Concave up when $H''(x) > 0$. Since $H''(x) = f'(x)$, $\boxed{H \text{ is concave up on } (9,12)}$

g) At what x-value does $H(x)$ achieve its maximum value? Justify your answer.

$H'(x) = 0$ when $f(x) = 0$ $\therefore$ this occurs at $x = 6$ & $x = 12$

$H'(x)$ is never undefined

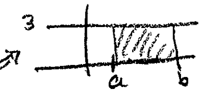
use a candidates test

x	0	6	12
H(x)	0	A	B

$H(6) > H(12)$ because $\int_6^{12} f(t)\,dt < 0$

$\therefore$ the maximum value of $H(x)$ is achieved when $x = 6$

11. [No Calculator] If $\int_a^b f(x)\,dx = a+2b$, then $\int_a^b [f(x)+3]\,dx =$ $\underbrace{\int_a^b f(x)\,dx}$ + $\int_a^b 3\,dx$



A $a+2b+3$

B $3b-3a$

C $4a-b$

$\boxed{\text{D} \quad 5b-2a}$

E $5b-3a$

$a+2b$ + $3(b-a)$

given

$= a+2b+3b-3a$

$= -2a+5b$

12. [No Calculator] Let $f(x) = \int_{-2}^{x^2-3x} e^{t^2}\,dt$. At what value of x is $f(x)$ a minimum?

A none

B 0.5

$\boxed{\text{C} \quad 1.5}$

D 2

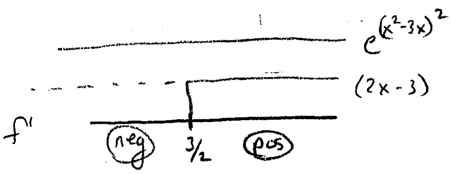
E 3

$f'(x) = e^{(x^2-3x)^2} \cdot (2x-3)$

$f'(x) = 0$ when $2x-3=0$ since $e^{(x^2-3x)^2} \neq 0$

so, $f'(x)=0$ when $x = 3/2$.

since $e^{(x^2-3x)^2} > 0$ for all values of x, the sign of f' depends solely on the $(2x-3)$ factor



since $f' < 0$ on $(-\infty, 3/2)$ and $f' > 0$ on $(3/2, \infty)$ f has a minimum at $x = 3/2$.

13. [Calculator] If $f(x) = \int_a^x \ln(2+\sin t)\,dt$, and $f(3) = 4$, what does $f(5) = ?$

$f(3) = \int_a^3 \ln(2+\sin t)\,dt = 4$

$f(5) = \int_a^5 \ln(2+\sin t)\,dt$

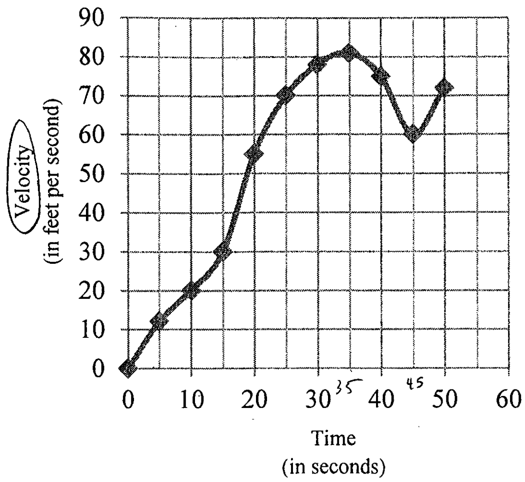
$\underbrace{\int_a^3 \ln(2+\sin t)\,dt}_{\text{given}} + \underbrace{\int_3^5 \ln(2+\sin t)\,dt}_{\text{calculate}} = \underbrace{\int_a^5 \ln(2+\sin t)\,dt}$

BTW... this holds true regardless of where a is!

$4 + .5550851972 = f(5)$

$\boxed{4.5550851972 = f(5)}$

Time (in seconds)	$v(t)$ (in ft/sec)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72



14. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \leq t \leq 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.

a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

$a(t) = v'(t)$ so when $v(t)$ is increasing, $a(t) > 0$

$a(t) > 0$ on $(0, 35) \cup (45, 50)$

b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \leq t \leq 50$.

average (Rate of change in velocity)

$$\frac{v(50) - v(0)}{50 - 0} = \frac{72 - 0}{50} = 14.4 \text{ ft/sec}^2$$

c) Approximate $\int_0^{50} v(t)\,dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

are 5, 15, 25, 35, 45

10

$$\int_0^{50} v(t)\,dt \approx \boxed{10(12) + 10(30) + 10(70) + 10(81) + 10(60)}$$

$$= 120 + 300 + 700 + 810 + 600$$

$$= 2530$$

This car has traveled 2530 feet from time = 0 to time = 50 seconds.