AP Calculus AB
Review 4.5 and 4.6

Name: KEY
Block: ________

1. You are interested in estimating the value of $\sqrt[4]{83}$. $\boxed{\text{NEED AN EQUATION}}$ $y=\sqrt[4]{x}=x^{1/4}$ & an x-value "close"

a) Use linearization. $y' = \frac{1}{4}x^{-3/4}$

Point $(81, 3)$

Slope $= y'(81) = \frac{1}{4}(81)^{-3/4} = \frac{1}{4}\cdot\frac{1}{27} = \frac{1}{108}$

Tangent Line $\boxed{y-3 = \frac{1}{108}(x-81)}$

$y = \frac{1}{108}(83-81)+3 = \frac{1}{54}+3 = \boxed{3\,1/54}$

$\therefore \sqrt[4]{83} \approx 3\,1/54$

b) Use differentials.

$dy = \frac{1}{4}x^{-3/4}dx$

$dy = \frac{1}{4}(81)^{-3/4}(2)$

$= \frac{1}{4}\cdot\frac{1}{27}\cdot\frac{2}{1}$

$= \frac{1}{54}$

since $\sqrt[4]{81} = 3$ … $\sqrt[4]{83} \approx 3\,1/54$

c) Find the error in your approximation.

$\sqrt[4]{83} - 3\frac{1}{54} \approx \boxed{-.000169039226}$

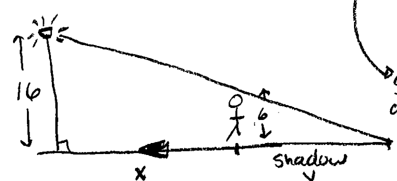
2. The Volume of a Sphere is given by $V = \frac{4}{3}\pi r^3$. Use differentials to estimate how much the volume will change when the radius is increased from 2 cm to 2.05 cm.

$dV = \frac{4}{3}\pi\cdot 3r^2 dr$ $\quad r=2,\ dr=.05$

$\therefore$ the Volume increases $\approx 2.513\ cm^3$

$dV = \frac{4}{3}\pi\cdot 3(2)^2(.05) \approx .8\pi \approx 2.513\ cm^3$

3. A 6 foot tall man walks at a rate of 5 ft/sec toward a streetlight that is 16 feet above the ground. At what rate is the length of his shadow changing when he is 10 feet from the base of the light?



Find dy/dt when $x=10$

$\frac{dx}{dt} = -5$

$\frac{6}{16} = \frac{y}{x+y}$

$6x+6y = 16y$

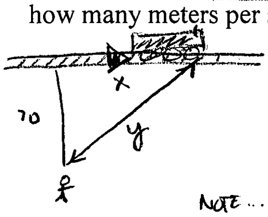
EQUATION: $6x = 10y$

$6\,dx/dt = 10\,dy/dt$

$6(-5) = 10\,dy/dt$

$\boxed{\frac{dy}{dt} = -3 \text{ ft/sec}}$

4. An observer 70 meters south of a railroad crossing watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?



$dx/dt = 60$

Find dy/dt when $x = 240$

$\frac{60m}{s}\cdot 4s = 240m$

$\boxed{\text{EQUATION:}}$ $70^2 + x^2 = y^2$

$2x\,dx/dt = 2y\,dy/dt$

$2(240)(60) = 2(250)\cdot dy/dt$

$\boxed{dy/dt = 57.6\ m/s}$

NOTE… when $x=240$, $(70)^2+(240)^2 = y^2 \Rightarrow y = 250$

5. A shadow of a 70 foot tall tree is cast on the ground as the sun goes down. If the shadow is increasing 2 ft/min, find how fast the angle of elevation from the tip of the shadow to the sun is changing when the shadow is 120 feet long? $dx/dt = 2$

Find $d\theta/dt$ when $x = 120$

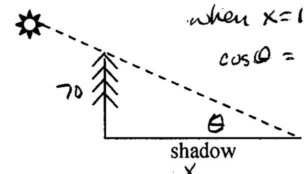
$\boxed{\text{EQUATION}}$ $\tan\theta = \frac{70}{x}$

OR $\tan\theta = 70x^{-1}$

$\sec^2\theta\,d\theta/dt = -70x^{-2}\,dx/dt$

$\frac{19300}{14400}\frac{d\theta}{dt} = -70(120)^{-2}(2)$

$\boxed{d\theta/dt = -\frac{7}{965}\ \text{Rad/sec}} \approx -.007$



when $x=120$, hyp $=\sqrt{19300}$

$\cos\theta = \frac{120}{\sqrt{19300}}$

$\therefore \sec^2\theta = \frac{19300}{14400}$

6. A dinghy is pulled toward a dock by a rope from the bow through a ring on the dock 6 feet above the bow as shown in the figure below. The rope is hauled in at the rate of 2 ft/sec. $dy/dt = -2$

a) How fast is the boat approaching the dock when 10 ft of rope are out? when $y=10$, $x=8$

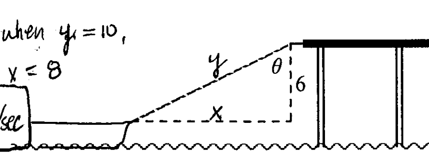
Find dx/dt

$\boxed{\text{EQUATION:}}$ $x^2+6^2 = y^2$

$2x\,dx/dt = 2y\,dy/dt$

$2(8)\,dx/dt = 2(10)(-2)$

$\boxed{\frac{dx}{dt} = -2.5\ \text{ft/sec}}$



b) At what rate is angle θ changing at that moment?

$\boxed{\text{EQUATION:}}$ $\cos\theta = \frac{6}{y}$

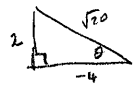
OR $\cos\theta = 6y^{-1}$

$-\sin\theta\cdot d\theta/dt = -6y^{-2}\,dy/dt$

$-\left(\frac{8}{10}\right)\cdot d\theta/dt = -6(10)^{-2}(-2)$

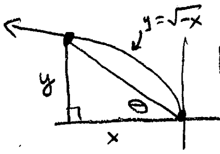
$\boxed{d\theta/dt = -.15\ \text{RADIANS/SEC}}$

NOTE: the x-coordinate is decreasing at a rate of 8 m/sec, but in the Δ that means $dx/dt = +8$



7. A particle moves from right to left along the parabolic curve $y=\sqrt{-x}$ in such a way that its x-coordinate (in meters) decreases at the rate of 8 m/sec. How fast is the angle of inclination θ of the line joining the particle to the origin changing when $x = -4$?

$\to dx/dt = 8$



$\boxed{\text{EQUATION:}}$ $\tan\theta = \dfrac{y}{x}$ But $y=\sqrt{-x}$ $\therefore \tan\theta = \dfrac{\sqrt{-x}}{x}$

when $x=-4$, $y=2$, & $\sec^2\theta = \frac{20}{16} = \frac{5}{4}$

$\sec^2\theta \cdot d\theta/dt = \dfrac{x\left[\frac{1}{2}(-x)^{-1/2}\cdot(-1)\frac{dx}{dt}\right] - (\sqrt{-x})(dx/dt)}{x^2}$

$\frac{5}{4}\cdot d\theta/dt = \dfrac{-4\left[\frac{1}{2}(4)^{-1/2}\cdot(-1)(+8)\right] - \sqrt{4}\cdot(+8)}{(-4)^2}$

$\boxed{d\theta/dt = -2/5 \text{ Rad/sec}}$

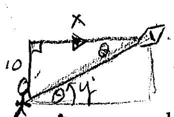
8. Charlie Brown is flying a kite (before it gets caught in the tree) at a height of 10 meters. The wind carries the kite horizontally away from him at a rate of 7 m/s.

a) How fast is the distance between Charlie Brown and the kite changing when he has let out 70 meters of string?

$dx/dt = 7$ m/s

Find dy/dt when $y=70$

$x = \sqrt{4800}$



$\boxed{\text{EQUATION:}}$ $x^2 + 10^2 = y^2$

$2x\,dx/dt = 2y\,dy/dt$

$2(\sqrt{4800})(7) = 2(70)\,dy/dt$

$\boxed{6.928 \approx dy/dt}$ m/s

b) How fast is the angle of elevation between Charlie Brown and the kite changing at the same moment?

when $y=70$

$\cos\theta = \dfrac{\sqrt{4800}}{70}$

$\Rightarrow \sec^2\theta = \dfrac{4900}{4800} = \dfrac{49}{48}$

$\tan\theta = \dfrac{10}{x}$

$\boxed{\text{EQUATION:}}$ $\tan\theta = 10x^{-1}$

$\sec^2\theta\; d\theta/dt = -10x^{-2}\cdot dx/dt$

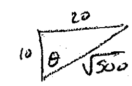
$\frac{49}{48}\cdot d\theta/dt = -10(\sqrt{4800})^{-2}\cdot(7)$

$\boxed{d\theta/dt \approx -.014 \text{ Rad/sec}}$

9. A security camera is centered along a 90 foot long hallway that is 10 feet wide. It is easiest to design the camera with a constant angular rate of rotation, but this results in a variable rate at which the images of the surveillance area are recorded. To resolve this issue, the system was designed with a variable rate of rotation so the camera will scan the hallway at a constant rate of 2 ft/sec.

$dx/dt = 2$ or -2 depending on direction camera is moving...

a) If you were standing 20 feet to the right of the camera, how fast is the camera rotating? (Does it matter which way the camera is moving?)



... when $x=20$, $\cos\theta = \frac{10}{\sqrt{500}}$ so, $\sec^2\theta = \frac{500}{100} = 5$

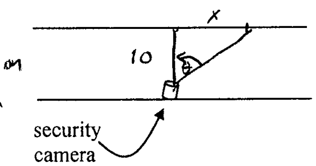
$\boxed{\text{EQUATION:}}$ $\tan\theta = \dfrac{x}{10}$ or $10\tan\theta = x$

$10\sec^2\theta\; d\theta/dt = dx/dt$

$10(5)\cdot d\theta/dt = 2$

$d\theta/dt = .04$ Rad/sec

or $-.04$ Rad/sec depending on which way the camera is moving.



b) If you are standing directly in front of the camera, how fast is the camera rotating?

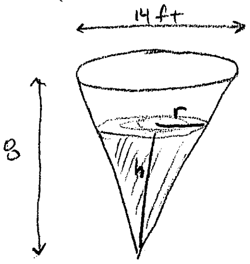
then $\theta = 0$ so... $\sec^2\theta = 1$

$10\sec^2\theta\,d\theta/dt = dx/dt$

$10(1)\dfrac{d\theta}{dt} = 2$

$d\theta/dt = .2$ Rad/sec (or $-.2$ Rad/sec)

10. A conical tank (with vertex down) is 14 feet across at the top and 8 feet deep. If water is flowing into the tank at a rate of 24 cubic feet per minute, find the rate of change of the depth of the water when the water is 3 feet deep. (The volume of a cone is $V=\frac{1}{3}\pi r^2 h$)



water is coming in at $\underbrace{24\ \text{ft}^3/\text{min}}_{dV/dt}$

Want to find dh/dt when $h=3$,

So write Volume in terms of h only

$V = \frac{1}{3}\pi r^2 h$

$\frac{r}{h} = \frac{7}{8}$

$r = \frac{7}{8}h$

$V = \frac{1}{3}\pi\left(\frac{7}{8}h\right)^2 h$

$V = \frac{1}{3}\pi\cdot\frac{49}{64}h^2\cdot h$

EQUATION: $\boxed{V = \dfrac{49\pi}{192}h^3}$

$dV/dt = \frac{49\pi}{192}\cdot 3h^2\,dh/dt$

$24 = \frac{49\pi}{192}\cdot 3(3)^2\cdot dh/dt$

$1.109 \approx \dfrac{dh}{dt}$

$\therefore$ the height of the water is increasing 1.109 ft/min