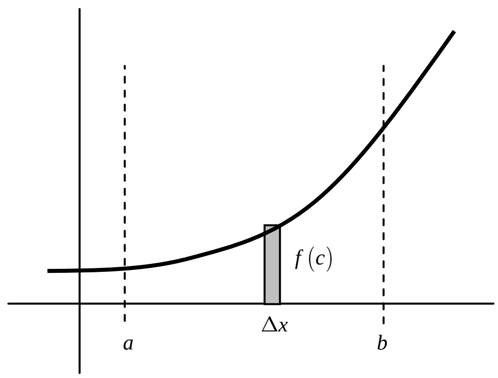
7.2 AREAS IN THE PLANE

Let's Review the concept of area as it relates to calculus!

Recall the area under a curve can be approximated through the use of Riemann sums: We can break the area into rectangles. Consider the one rectangle drawn. It's height is given by the function value of the curve at the right endpoint and the width is given as Δx. The area under the curve then is approximately the sum of the areas of ALL the rectangles just like this one.



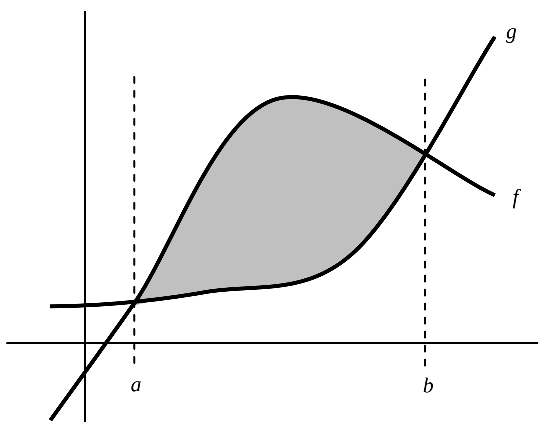
$$\text{Area} \approx \sum_{k=1}^{n} f(c_k)\Delta x_k$$

As the number of rectangles, *n*, increases, the approximated area gets closer to the actual area, so we say

$$\text{Area under the curve} = \lim_{n\to\infty}\sum_{k=1}^{n} f(c_k)\Delta x_k = \int_a^b f(x)\,dx$$

We can apply this same concept to the *area between curves*. Consider the two functions *f* and *g* below.

Example 1: Draw a rectangular strip. What is the height and width of your rectangle? Would the height and width of the rectangle strip be different if you drew it in a different place?



Example 2: The area between the curves is approximately the sum of all of these rectangles. We can write this as

Example 3: How can we get closer to the ACTUAL area between the curves?

Example 4: If we let the number of rectangles approach infinity, then we have

Area of a Region Between Two Curves

If *f* and *g* are continuous on [a, b] and $g(x) \le f(x)$ for all *x* in [a, b], then the area of the region bounded by the graphs of *f* and *g* and the vertical lines *x* = *a* and *x* = *b* is

$$A = \int_a^b \left[f(x) - g(x) \right] dx$$

Example 5: Find the area of the region bounded by the graphs of $y = x^2 + 2$, $y = -x$, *x* = 0, and *x* = 1.

Step 1: Draw a picture and shade the desired region.

Step 2: Draw an arbitrary rectangular strip.

Step 3: Using the area of the rectangular strip as a guide, set up and solve an integral to find the area between the curves.

Example 6: Find the area of the region bounded by the graphs of $x = 3 - y^2$ and $x = y + 1$.