7.1 INTEGRAL AS NET CHANGE

Distance versus Displacement

We have already seen how the position of an object can be found by finding the integral of the velocity function. The change in position is a *displacement*. To see the difference between distance and displacement, consider the following saying:

"Two steps forward and one step back"

Example 1: What is the total distance traveled? _________ What is the total displacement? _________

To find displacement, we only need to find $\int_a^b v(t)\,dt$.

In order to find your new location, we say that your *new position* = *initial position* + *displacement*.

To find total distance we use $\int_a^b |v(t)|\,dt$ or find when the object is moving in the negative direction, break the integral into pieces and subtract the value of the integral for the area under the curve.

Example 2: Suppose the velocity of a particle moving along the x – axis is given by $v(t) = 6t^2 - 18t + 12$ when $0 \le t \le 2$.

a) When is the particle moving to the right? When is it moving left? When is it stopped?

b) Find the particle's displacement for the time interval.

c) Find the particle's total distance traveled by setting up ONE integral and using your calculator.

d) Find the particle's total distance traveled without using absolute value.

Consumption over Time

Velocity is not the only rate in which you can integrate to get a total. In fact if you were given a function that gave the number of tickets per hour that the police wrote each day, and you wanted to find the total number of tickets in a 24-hour period, you could integrate.

Example 3: The tide removes sand from Sandy Point Beach at a rate modeled by the function *R* given by

$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function *S*, given by

$$S(t) = \frac{15t}{1+3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

a) How much sand will the tide remove from the beach during this 6-hour period?
Indicate units of measure.

b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t.

c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

d) For $0 \le t \le 6$, at what time t is the amount of sand on the beach a minimum?
What is the minimum value? Justify your answers.