6.1 ANTIDERIVATIVES AND SLOPE FIELDS

Notecards from Section 6.1: Slope Fields (Revisited); Indefinite Integrals

Indefinite Integrals

In the previous chapter we dealt with definite integrals. Definite integrals had limits of integration . Indefinite integrals do not.

The <u>set</u> of all antiderivatives of a function $f(x)$ in the **indefinite integral of *f* with respect to *x*** and is denoted

$$\int f(x)\,dx$$

Recall, that all antiderivatives differ by a constant, so if $F'(x) = f(x)$, then $\int f(x)\,dx = F(x) + C$, where C is the constant of integration. The following table gives a list of results you should already be familiar with.

Integral Formulas

1. Power Rule for $n \neq -1$: $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$
2. Rule for $n = -1$: $\int \frac{1}{x}\,dx = \ln|x| + C$
3. $\int e^{kx}\,dx = \frac{e^{kx}}{k} + C$
4. $\int \sin(kx)\,dx = \frac{-\cos(kx)}{k} + C$
5. $\int \cos(kx)\,dx = \frac{\sin(kx)}{k} + C$
6. $\int \sec^2(x)\,dx = \tan(x) + C$
7. $\int \csc^2(x)\,dx = -\cot(x) + C$
8. $\int \sec(x)\tan(x)\,dx = \sec(x) + C$
9. $\int \csc(x)\cot(x)\,dx = -\csc(x) + C$
10. $\int \frac{dx}{x^2+1} = \tan^{-1}(x) + C$
11. $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}(x) + C$
12. $\int a^x\,dx = \frac{a^x}{\ln a} + C$ … where a is a constant

Example 1: Evaluate each integral.

a) $\int \left(-x^{-3} + \sqrt[3]{x} - 1 + e^{3x}\right) dx$

b) $\int (3\sin x - \sin 3x)\,dx$

Differential Equations

A **differential equation** is an equation containing a derivative. Just like in Algebra, when you want to solve an equation, you use an inverse operation. To "undo" a derivative we take an ____________________________________.
Recall, that a function can have many antiderivatives, all of which vary by a _________________________.

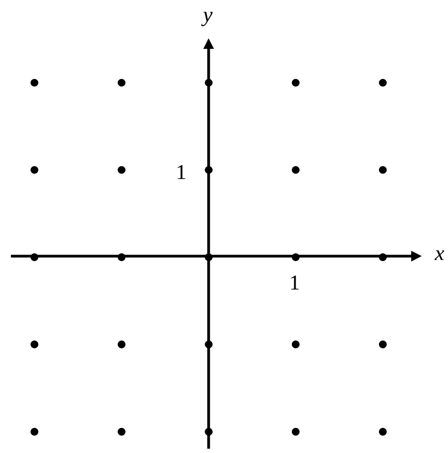
Solving a differential equation involves finding a unique equation that satisfies some *initial conditions* or *initial values*.
The **order** of a differential equation is the order of the highest derivative involved in the equation.

Example 2: Solve $\frac{dy}{dx} = \sin x$ by **separation of variables** if $y(0) = 2$.

A Graphical Look at Differential Equations

A **slope field** (or direction field) for the first order differential equation $\frac{dy}{dx} = f(x, y)$ is a plot of short line segments with slope $f(x, y)$ for a lattice of points (x, y) in the plane.

Example 3: On the diagram below, plot the slope field of the differential equation $\frac{dy}{dx} = 2y$.



Example 4: Suppose that you know that the point (0, –1) is on a particular solution of the differential equation above. By following slopes, draw on the diagrams above what you think the particular solution look like. (♬: The graph should follow the pattern of the slope field, but may go between the points rather than through them.)

Example 5: Solve the differential equation $\frac{dy}{dx} = 2y$ from the previous example by first **separating the variables**. Find the particular solution that contains the point given in the last example. Does your solution make sense when compared to the graph of the slope field?