4.5 LINEARIZATION

Notecards from Section 4.5: Linearization; Differentials; Absolute vs Relative vs Percentage Change

Linearization

The goal of linearization is to approximate a curve with a line. Why? Because it's easier to use a line than a curve!
**All you have to do is find the equation of a tangent line and use the tangent line instead of the original function.

Example 1: Consider $f(x) = \sqrt{x}$. We all know that $f(4) = 2$, but without a calculator, how can we find $f(4.1)$?

a) Find the equation of the tangent line for $f(x)$ when $x = 4$. [Your book refers to this as $L(x)$.]

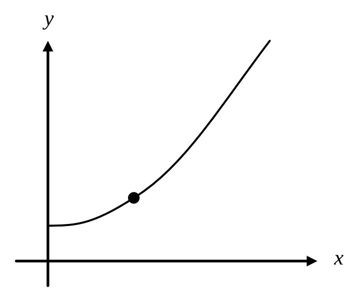
b) The tangent line you found is <u>approximately the same as $f(x)$ "centered at $x = 4$"</u>.
Use your tangent line to approximate $f(4.1)$.

c) Use a calculator to approximate $f(4.1)$. How close is the approximation?

Differentials

Approximations aren't exact! (Aren't you glad you woke up this morning to hear that enlightening bit of information?!) If we use a tangent line to approximate a curve, it gives us a good estimate, <u>as long as we don't go too far away from the center point</u>. Wouldn't it be nice if we knew how far off our approximation is going to be?

Example 2: Consider the function f pictured to the right.



a) Label the center point $(c, f(c))$, and draw the tangent line at that point.

b) What is the equation of the *tangent line* you drew?
Keep in mind … this is just a linearization of the curve.

c) Move Δx to the right of c. What is the y-value of this new point on the tangent line?

d) The ACTUAL change in y, Δy, is given by ____________________.

When Δx is very small (infinitesimally small), we say that $\Delta x = dx$ (the differential of x).

e) We will denote the APPROXIMATE change in y with dy. Find dy.

What differentials allow us to do is treat the *dy* and *dx* terms of the notation $\frac{dy}{dx}$ as separate terms.

Finding a differential is simply finding a derivative using different notation. JUST REMEMBER THAT …
dy is a small change in *y*, and *dx* is a small change in *x*.

Example 3: Find the differential *dy* when *dx* = 0.01 and *x* = 2, if $y = x^5 - 4x^3$. *Explain* what you've found.

Percentages & Errors

When you have been asked to find a percentage change, you always find the "change" divided by the original amount.

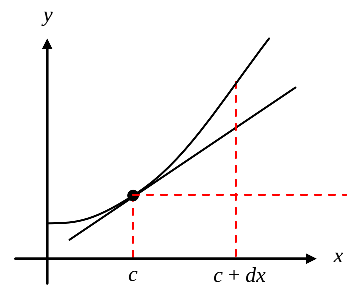
"Approximate percent change in *y*" can be found by $\frac{dy}{y}$, since *dy* is the approximate change in *y*.

When you make a mistake in a measurement and then use that measurement to calculate something else, the resulting effect the original error has on the calculation can be found using differentials. The "errors" will be the "change".

"Approximate errors" will be found using differentials
dx = approximate "error" in the measurement of *x*
dy = approximate "error" in the measurement of *y*

"Actual errors" are found by finding the change in the function values (Δy).
$f(c+dx) - f(c)$

"Errors in an approximation" … (i.e. how "close" was the approximation)
(Actual Change in Function) – (Approximate Change in Function)
$\Delta y - dy$



Example 4: Use differentials to estimate $\sqrt[3]{29}$. Find the error in your approximation.

Example 5: The measurement of a side of a square is found to be 15 centimeters. The possible error in measuring the side is 0.05 centimeter.

a) Find the approximate error in computing the area of the square.

b) Approximate the percent error in computing the area of the square.

c) Estimate the maximum allowable percent error in measuring the side if the error in computing the area cannot exceed 2.5%