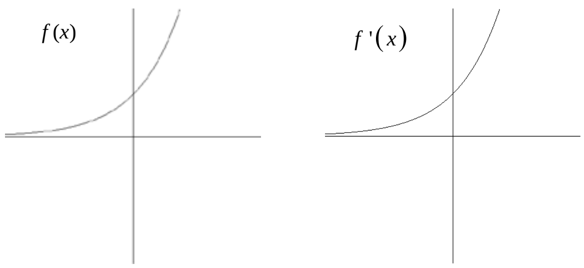
3.9 DERIVATIVES OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS

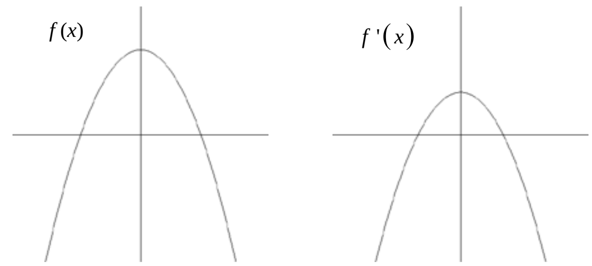
Notecards from Section 3.9: Derivatives of Exponential Functions, Derivatives of Logarithmic Functions, Logarithmic Differentiation

Example 1: Which of the following pairs of graphs could represent the graph of a function AND its derivative?

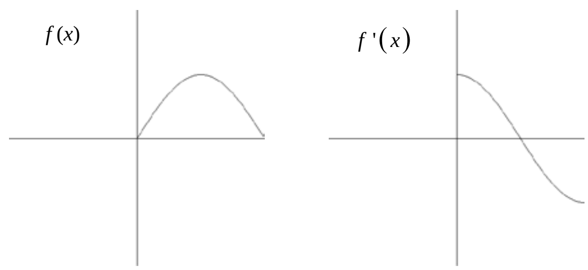
I.



II.



III



A) I only B) II only C) III only D) I and III only E) II and III only

Now for the easiest derivative rule of the year … notice the first pair of graphs (I) above.

Derivative of $f(x) = e^x$

$$\frac{d}{dx}\left[e^x\right] = e^x$$

The proof of this can be done using the definition of a derivative.

The Chain Rule and $f(x) = e^x$

If u is a differentiable function of x, then

$$\frac{d}{dx}\left[e^u\right] = e^u \cdot \frac{du}{dx}.$$

Example 2: Find $\frac{d}{dx}\left[e^{2x-1}\right]$

Example 3: Find $\frac{d}{dx}\left[e^{-3/x}\right]$

The inverse of an exponential function is the natural logarithm function.

Derivative of $f(x) = \ln x$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

Example 4: Prove the derivative rule above using implicit differentiation.

The Chain Rule and $f(x) = \ln x$

If u is a differentiable function of x, then

$$\frac{d}{dx}[\ln u] = \frac{1}{u} \cdot \frac{du}{dx} \qquad \ldots \text{ or } \ldots \qquad \frac{d}{dx}[\ln u] = \frac{u'}{u}$$

Example 5: Let $y = \ln(2x+2)$. Find y'.

Example 6: Let $f(x) = \ln(\tan x)$. Find $f'(x)$

Logarithmic Differentiation

We can use the properties of logarithms to simplify some problems. Here's a quick refresher on those properties.

Definition of a logarithm: $\log_b a = c \quad \Leftrightarrow \quad b^c = a$

Change of Base Formula: $\log_b a = \dfrac{\log a}{\log b}$ or $\dfrac{\ln a}{\ln b}$

3 Rules of Logarithms:

1. $\log_b(MN) = \log_b(M) + \log_b(N)$
2. $\log_b\left(\dfrac{M}{N}\right) = \log_b(M) - \log_b(N)$
3. $\log_b(M^k) = k \cdot \log_b(M)$

Example 7: Use the properties of logarithms to rewrite the function, then find the derivative of $y = \log_5 \sqrt{x}$.

Example 8: Use the technique of logarithmic differentiation to find $\frac{dy}{dx}$ for $y=\frac{x\sqrt{x^2+1}}{(x+1)^{2/3}}$.

By utilizing the rules of logarithms and implicit differentiation, you can turn an exponential equation into an equation involving logarithms that is usually easier to deal with.

Example 9: Find $\frac{dy}{dx}$ if $y=2^x$.

Example 10: Find $\frac{dy}{dx}$ if $y=3^x$.

The previous two examples lead us to the following result.

Derivative of a^u … *a* is a consant

If $a>0$ and $a\neq 1$ and *u* is a differentiable function of *x*, then

$$\frac{d}{dx}\left[a^u\right]=\ln a\cdot a^u\,\frac{du}{dx}$$

Example 11: Find the derivative of the function $g(x)=e^{5x}+7^x+\ln\left(x^3+4\right)$