3.8 DERIVATIVES OF INVERSE TRIGONOMETRIC FUNCTIONS

Notecards from Section 3.8: Derivatives of Inverse Trig Functions, Derivative of an Inverse Function at a Point (p, q)

Derivatives of Inverse Trigonometric Functions

One of the things in the AP Calculus AB Course Description is "Use of implicit differentiation to find the derivative of an inverse function". With that said, let's find the derivative of the inverse sine function using implicit differentiation.

Example 1: Suppose $y = \sin^{-1} x$. Find $\frac{dy}{dx}$ using implicit differentiation.

Derivatives of Inverse Trigonometric Functions where u is a function of x.

1. $\frac{d}{dx}\left[\sin^{-1}(u)\right] = \frac{u'}{\sqrt{1-u^2}}$
2. $\frac{d}{dx}\left[\cos^{-1}(u)\right] = \frac{-u'}{\sqrt{1-u^2}}$
3. $\frac{d}{dx}\left[\tan^{-1}(u)\right] = \frac{u'}{1+u^2}$
4. $\frac{d}{dx}\left[\cot^{-1}(u)\right] = \frac{-u'}{1+u^2}$
5. $\frac{d}{dx}\left[\sec^{-1}(u)\right] = \frac{u'}{|u|\sqrt{u^2-1}}$
6. $\frac{d}{dx}\left[\csc^{-1}(u)\right] = \frac{-u'}{|u|\sqrt{u^2-1}}$

* Domains are restricted to make them functions

* $\sin^{-1}(x)$ and $\arcsin(x)$ are the same thing

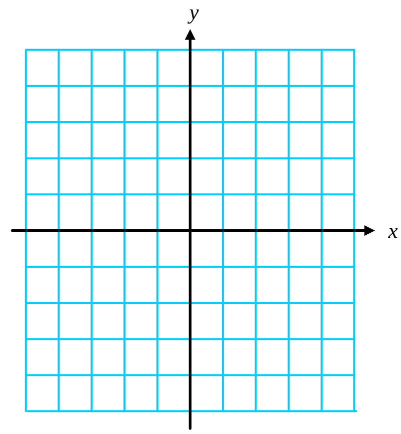
Again … you should commit these to memory as quickly as possible!

Example 2: Find the derivative of $f(t) = \sin^{-1}\left(t^2\right)$

Example 3: Find the derivative of $y = \tan^{-1}\left(\sqrt{x-1}\right)$

Derivatives of Other Inverse Functions

Example 4: Graph the line $y = 4x + 1$



a) What is the slope of the line?

b) Graph the inverse function.

c) What is the slope of the inverse function?

d) If (2, 9) is on the original line, what point does it correspond to on the inverse function?

The slope of the line at (2, 9) on the original function is the ________________ of the slope of the inverse. The difference is that the slope of the inverse is calculated using the point ____________ instead of (2, 9).

New Notation

We have a different notation is used to describe the derivative of f at the point $x = a$. We can write

$$\left.\frac{df}{dx}\right|_{a}$$

Using the concepts from the last example, we can find the derivative of the inverse function without actually finding the inverse function. We will use the notation above to describe the relationship.

Derivative of the inverse function at a point (p, q) … this implies the point (q, p) is on the original function.

To find the derivative of f^{-1} at the point (p, q) we find the reciprocal of the derivative of f at the point (q, p).

$$\left.\frac{df^{-1}}{dx}\right|_{p} = \frac{1}{\left.\frac{df}{dx}\right|_{q}}$$

♬ : In other words, if f and g are inverses, then their derivatives at the inverse points are reciprocals.

Example 5: Let $f(x) = x^5 + 2x - 1$. Verify (0, –1) is on the graph. Find $\left(f^{-1}\right)'(-1)$.

Example 6: Let $f(x) = x^3 + 2x - 1$. Find $\left.\frac{df^{-1}}{dx}\right|_{2}$.