3.5 DERIVATIVES OF TRIGONOMETRIC FUNCTIONS

Notecards from Section 3.5: Derivatives of 6 Trig Functions

The goal for this lesson is to introduce the derivatives of the 6 trigonometric functions. It is vital to your success in this course that you commit all 6 of these to memory *as quickly as possible*!

Using the derivatives of sin *x* and cos *x*, you can find the derivatives of the other 3 trig functions as well. The remaining portion of these notes will show how to use the definition of a derivative to find the derivatives of the sine function, followed by a few examples using these 6 rules along with the power, product, and quotient rules.

Derivatives of the Six Basic Trigonometric Functions

$\frac{d}{dx}[\sin x] = \cos x$ $\frac{d}{dx}[\cos x] = -\sin x$ $\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\cot x] = -\csc^2 x$ $\frac{d}{dx}[\sec x] = \sec x \tan x$ $\frac{d}{dx}[\csc x] = -\csc x \cot x$

First, to prove the derivative of sine, we need a few background bits of information …

You should already know this limit … $\lim_{x\to 0}\frac{\sin x}{x}$ = ____.

Example 1: Investigate $\lim_{x\to 0}\frac{\cos x - 1}{x}$.

Example 2: To prove this algebraically, multiply the top and bottom by $(\cos x + 1)$, then evaluate the limit.

Example 3: Find the derivative of sin *x* using the limit definition of the derivative.

You can prove $\frac{d}{dx}[\cos x] = -\sin x$ using the same method and the same two limits above.

Example 4: Find the derivative of tan *x* using the quotient rule and the derivatives of sin *x* and cos *x*.

We now have the power rule, the product rule, the quotient rule, and the derivatives of all 6 trig functions at our disposal.

Example 5: Find the derivative of each function. Before you begin, state which rule(s) you are going to have to use. The product rule seems to be the rule that people forget to use … try not to be one of those people! ☺

a) $f(x) = x^2 \sin x$

b) $f(x) = \dfrac{\cos x}{x}$

c) $g(t) = \sqrt{t} + 4\sec t$

d) $h(\theta) = 5\sec\theta + \tan\theta$

e) $h(s) = \dfrac{1}{s} - 10\csc s$

f) $y = x\cot x$

Using the TI – 83+

You can use your calculator to graph the derivative for you using the procedures outlined below. This is not a required skill for success in this course, just something else your calculator can do. IF you have a TI-84, this process will be much quicker. I will leave this page for you to read if you are interested.

We will be using the `nDeriv(` function, except we will be using it to define a function under Y_1. Remember the syntax is

`nDeriv(function,variable,value)`

For our example, let's use cos *x*. The difference will be that our function will be entered as a function of any variable other than *x*, and differentiated with respect to that same variable, and evaluated at the value of *x* instead of an actual number.

So, we want to enter the following:

Y_1 = `nDeriv(cos(T),T,X)`

Step 1: Press [Y=].

Step 2: To enter `nDeriv(` … Press [MATH] then 8: `nDeriv(`

Step 3: Enter the function using the [ALPHA] key to enter a variable other than *x*. I chose *T*, but it should work with any letter you choose.

Step 4: After entering a comma, enter the same letter from step 3 as the variable you want to take the derivative with respect to. Again, I chose *T*, it's completely your choice.

Step 5: Enter the last comma, and then push the [X,T,Θ,*n*] button to enter the *X*. Do NOT use the [ALPHA] key to enter the *X* or it will not work.

Step 6: Graph the function by pressing [GRAPH].

Example: Graph the derivative of $f(x) = \ln x$. What function does this look like? Graph your guess on the same screen.

Example: Graph the derivative of $f(x) = e^x$. What function does this look like? Graph your guess on the same screen.