3.4 VELOCITY AND OTHER RATES OF CHANGE

Notecards from Section 3.4: Relationships between Position, Velocity, and Acceleration; Velocity vs. Speed

Instantaneous Rates of Change

We have already seen that the instantaneous rate of change is the same as the slope of the tangent line and thus the derivative at that point. Unless we use the phrase "average rate of change", we will assume that in calculus the phrase "rate of change" refers to the instantaneous rate of change.

Example 1: The length of a rectangle is given by $2t + 1$ and its height is $\sqrt{t}$, where t is time in seconds and the dimensions are in centimeters. Find the rate of change of the area with respect to time, and indicate the units of measure for this rate.

Motion along a line

When a spring attached to a wall is stretched and then released, it moves back and forth. This motion can be described using functions involving sine and cosine (which we are not ready for just yet …). However, motion along a line in other circumstances (either horizontal or vertical lines) can also be described using functions. Typically, we use a position function $s(t)$ to describe the position s of an object after t seconds.

Relationships between Position, Velocity, and Acceleration

The *displacement* of an object is the TOTAL CHANGE IN POSITION.

The *average velocity* of the object is described as TOTAL CHANGE IN POSITION (displacement) divided by the TOTAL CHANGE IN TIME. It can be thought of as the slope of the line connecting two points on a position function.

The *instantaneous velocity* of the object is the *derivative of the position function*. Unless term "average velocity" is used, we will assume velocity refers to instantaneous velocity. It is the slope of a tangent line to the position function.

Positive Velocity indicates movement in the positive direction.
Negative velocity indicates movement in the negative direction.

Speed is the absolute value of velocity. Thus speed is always positive value, whereas, ***velocity indicates direction***.

Acceleration is the rate of change in velocity, implying then that acceleration is the *derivative of velocity*. Since it is the derivative of velocity, it is also the *second derivative of position*.

Example 2: Bugs Bunny has been captured by Yosemite Sam and forced to "walk the plank". Instead of waiting for Yosemite Sam to finish cutting the board from underneath him, Bugs finally decides just to jump. Bugs' position, s, is given by $s(t) = -16t^2 + 16t + 320$, where s is measured in feet and t is measured in seconds.

a) What is Bugs' displacement from $t = 1$ to $t = 2$ seconds?

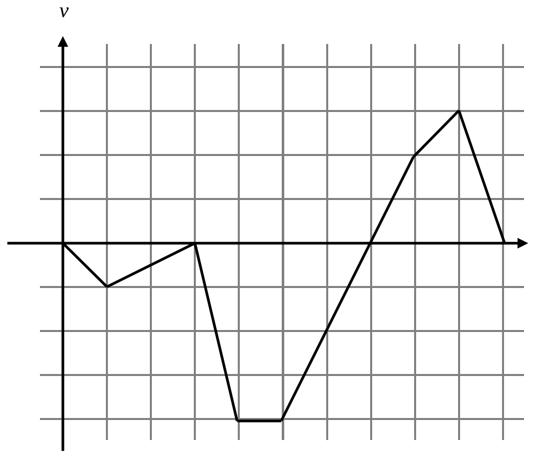
b) When will Bugs hit the ground?

c) What is Bugs' velocity at impact? (What are the units of this value?)

d) What is Bugs' speed at impact?

e) Find Bugs' acceleration as a function of time. (What are the units of this value?)

Example 3: Suppose the graph below shows the velocity of a particle moving along the x – axis.
Justify each response.



a) Which way does the particle move first?

b) When does the particle stop?

c) When does the particle change direction?

d) When is the particle moving left?

e) When is the particle moving right?

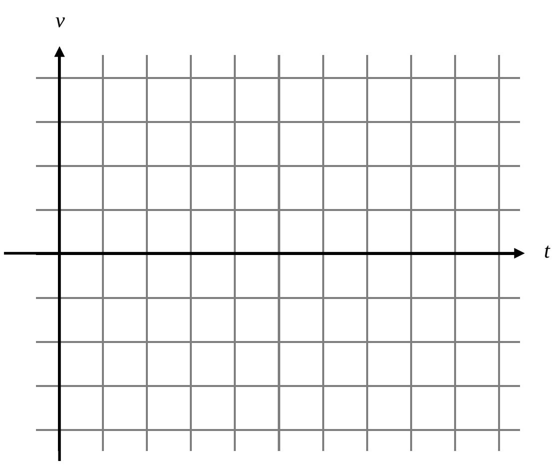
f) When is the particle speeding up?

g) When is the particle slowing down?

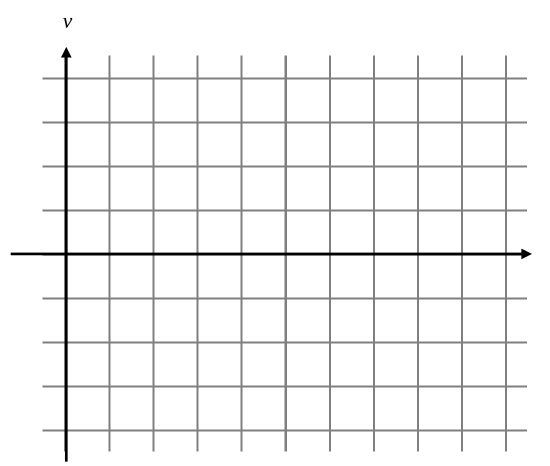
h) When is the particle moving the fastest?

i) When is the particle moving at a constant speed?

h) Graph the particle's acceleration for $0 < t < 10$.

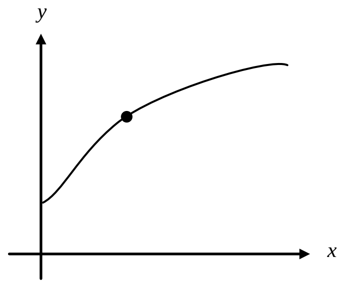


j) Graph the particle's speed for $0 \leq t \leq 10$.



Derivatives in Economics

Economists use calculus to determine the rate of change of costs with respect to certain factors. *Example*: Draw a tangent line at the indicated point on the function below.



Suppose the original function was a profit function. Can we use the tangent line you drew to estimate the profit?

Label the initial point (*x*, *P* (*x*)). If we increased *x* by 1, what is the actual change in the profit?

Now consider the tangent line …

The slope of the tangent line is given by ____________, and slope of a line is given by _________.

If we consider a change in *x* of 1, then the slope of the tangent line = the change in *y* …. ($P'(x) = \Delta y$).

This is the underlying principle in the following definitions. If *x* changes by 1 unit, then the change in *y* is approximately the value of the derivative at the original *x*.

Marginal Cost: The marginal cost at *x*, given by $C'(x)$, is the approximate cost of the (*x* + 1)st item.

Marginal Revenue: The marginal revenue at *x*, given by $R'(x)$, is the approximate revenue generated by the (*x* + 1)st item.

Marginal Profit: The marginal profit at *x*, given by $P'(x)$, is the approximate profit generated by the (*x* + 1)st item.

Example 4: Suppose that the daily cost, in dollars, of producing *x* radios is $C(x) = 0.002x^3 + 0.1x^2 + 42x + 300$, and currently there are 40 radios produced daily.

a) What is the current daily cost?

b) What would the actual additional daily cost of increasing production to 41 radios daily?

c) What is the marginal cost of the 41^{st} unit?

Example 5: Suppose the cost of producing x units is given by $c(x) = 4x^2 + \frac{300}{x}$. What is the marginal cost of producing the 11^{th} unit?