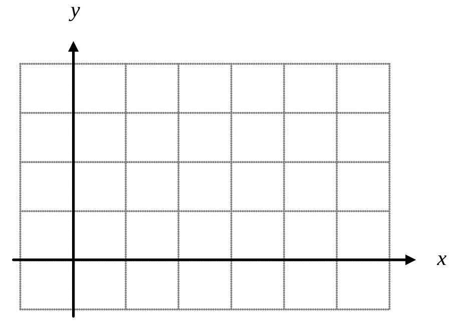
3.2 DIFFERENTIABILITY

Notecards from Section 3.2: Where does a derivative NOT exist, Definition of a derivative (3rd way).

The focus on this section is to determine when a function fails to have a derivative. For all you non-English majors, the word *differentiable* means you are able to take a derivative, or the derivative exists.

Example 1: Using the grid provided, graph the function $f(x) = |x-3|$.



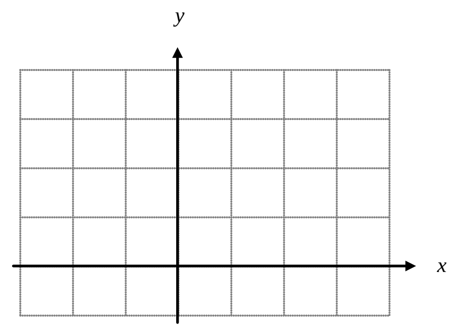
a) What is $f'(x)$ as $x \to 3^-$?

b) What is $f'(x)$ as $x \to 3^+$?

c) Is f continuous at $x = 3$?

d) Is f differentiable at $x = 3$?

Example 2: Graph $f(x) = x^{2/3}$

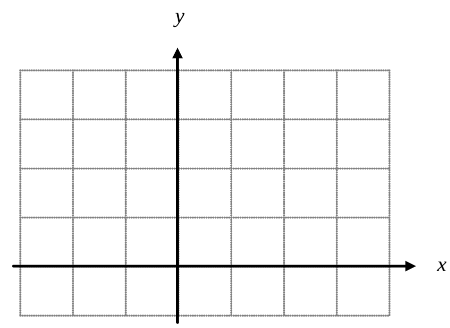


a) Describe the derivative of $f(x)$ as x approaches 0 from the left and the right.

b) Suppose you found $f'(x) = \dfrac{2}{3\sqrt[3]{x}}$.

What is the value of the derivative when $x = 0$?

Example 3: Graph $f(x) = \sqrt[3]{x}$



a) Describe the derivative of $f(x)$ as x approaches 0 from the left and the right.

b) Suppose you found $f'(x) = \dfrac{1}{3\sqrt[3]{x^2}}$.

What is the value of the derivative when $x = 0$?

These last three examples (along with any graph that is not continuous) are *NOT differentiable*. The first graph had a "*corner*" or a sharp turn and the derivatives from the left and right did not match. The second graph had a "*cusp*" where secant line slope approach positive infinity from one side and negative infinity from the other. The third graph had a "*vertical tangent line*" where the secant line slopes approach positive or negative infinity from both sides.

The first two can be referred to as a "pointy place" .

In all three of the previous examples the functions were continuous, but failed to be differentiable at certain points.

Continuity does not guarantee differentiability, but it does work the other way around.

Differentiability Implies Continuity

If f is differentiable at $x = c$, then f is continuous at $x = c$.

Example 4: For the logical statement … if A, then B … the converse is written … if B, then A.
The converse of the statement in the box is NOT true! What is the converse?

Example 5: The contrapositive of any statement is logically equivalent to the original statement. For the logical statement … if A, then B … the contrapositive is written … if not B, then not A. What is the Contrapositive to the statement in the box?

Example 6: If you are given that f is differentiable at $x = 2$, then explain why each statement below is true?

a. $\lim_{x\to 2} f(x)$ exists

b. $\lim_{h\to 0} \dfrac{f(2+h)-f(2)}{h}$ exists.

c. $f(2)$ exists

d. $\lim_{x\to 2} \dfrac{f(x)-f(2)}{x-2}$ exists.

Example 7: If f is a function such that $\lim_{x\to -3} \dfrac{f(x)-f(-3)}{x+3} = 2$, which of the following must be true?

A) The limit of $f(x)$ as x approaches –3 does not exist.

B) f is not defined at $x = -3$.

C) The derivative of f at $x = -3$ is 2.

D) f is continuous at $x = 2$.

E) $f(-3) = 2$

Using the TI – 83+

Most graphing calculators can take derivatives at certain points. In fact, it is necessary on the AP exam that you have a calculator that will take the derivative at a given point. However, they use a different method of calculating the derivative than our earlier definitions.

♬: The TI-89 will actually find the derivative formula, but when a derivative is needed on the calculator portion it has been asked only to evaluate the derivative at a point, thus removing the advantage of having a TI-89 over a TI-83 (or 84).

Example 8: We used the following formula to find the derivative in the last section. Provide a geometric interpretation (picture) of this formula:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Example: Here's a third way to define the derivative. Draw a picture to represent this formula:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}$$

Your graphing calculator uses the concept of this last definition to calculate derivatives.

To use your graphics calculator to find the derivative, use the `nDeriv(` function on the TI-83+. To access this function press [MATH], then 8 (or use [▲] and [▼] to go to `nDeriv(` and press [ENTER]). The `nDeriv(` function works as follows:

`nDeriv(function,variable,value)`

Where "*function*" is the function you want to find the derivative of, "*variable*" is the variable you are differentiating with respect to (usually *x*), and "*value*" is the point at which you want to find the derivative.

♬: Many times it is easier to type the function into `Y1`, and then enter `nDeriv(Y1,x,#)`.

Example: Use your calculator to find the derivative of $f(x) = x^2 - 3x + 2$ at $x = -3$. Express your answer with the correct notation.

Example: Use your calculator to find the derivative of the three examples at the beginning. What problems do you find? Why?

You can also find the derivative of your function from the graph.

Example: Find the derivative of $f(x) = x^2 - 3x + 2$ at $x = -3$ from the graph by following these steps.

Press [Y=] , and enter the equation.
Press [GRAPH].
Press [2nd] , [TRACE] … (Calc)
Press 6: … (dy/dx)
Press –3 … (this defines $x = -3$ and the calculator calculates the derivative at $x = -3$)

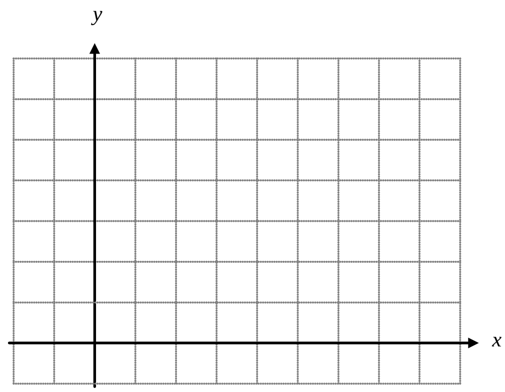
Express your answer on paper using the correct notation _____________________.

Example: Find the derivative of $f(x) = \left|x^2 - 3x + 2\right|$ at $x = 2$.

a) Using nDeriv(

b) Using the [GRAPH] screen.

c) Graph the function by hand on the grid below. Based on your graph, what is $f'(2)$?



Example: Find the derivative of $f(x) = \dfrac{1}{x}$ at $x = 0$.

(a) Using your calculator.

(b) Using any of the three definitions of a derivative.

Example: Using your calculator, find the equation of the tangent line to the graph of $f(x) = x^3 + x^2$ when $x = 2$. Show your work using correct notation.