3.1 DERIVATIVE OF A FUNCTION

Notecards from Section 3.1: Definition of a Derivative (2 of the 3 ways), Definition of the existence of a derivative at $x = c$ and at an endpoint, Slope Fields

In the last chapter we used a limit to find the slope of a tangent line. Without knowing it, you were finding a derivative all along. A derivative of a function is one of the two main concepts from calculus. The other is called an integral, and we will not get to that until later. The only change from the limit definition we used before, is that we are going to treat the derivative as a function derived from *f*.

Definition of a Derivative

The **derivative** of a function f with respect to the variable x is the function f' whose value at x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

provided the limit exists.

Anywhere that the derivative exists, we say that the function is *differentiable*.

Thus the derivative is a function that gives the slope of the function at any point.

Example 1: Other notation used to denote the derivative (we will use most of these).

Example 2: Use the definition of the derivative to find $f'(x)$.

a) $f(x) = 3x + 2$

b) $f(x) = x^3 + x^2$

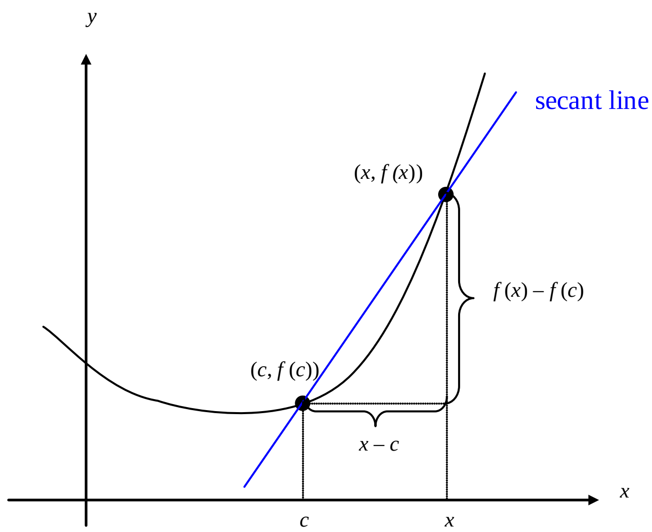
c) $f(x) = \dfrac{1}{\sqrt{x}}$

Alternative Definition #1 of the Derivative

An alternative definition of the derivative of *f* at *c* is

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

provided this limit exists.



♬: What this alternative definition allows us to do is to examine the behavior of a function as *x* approaches *c* from the left or the right. The limit exists (and thus the derivative) as long as the left and right limits exist and are equal.

Example 3: Use the alternative definition on the same examples as before.

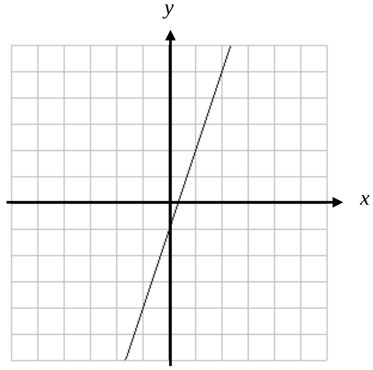
a) $f(x) = x^3 + x^2$

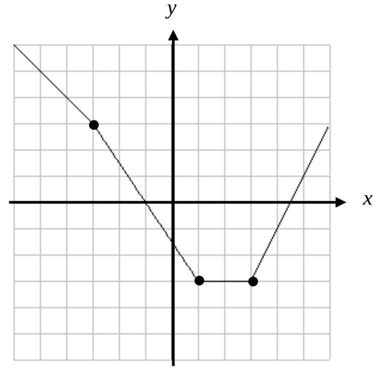
b) $f(x) = \frac{1}{\sqrt{x}}$

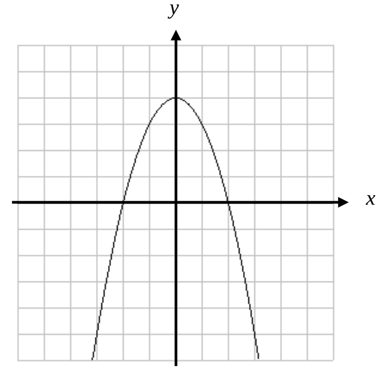
Relationships between the Graphs of f and $f\,'$.

Since a derivative at any point is equivalent to the slope of the function at that point, we can *estimate* what the original function looks like when we are given the graph of the derivative and vice – versa.

Example 4: Given the graph of *f*, sketch the graph of the derivative on the same set of axes.







For a few other examples … see http://people.hofstra.edu/stefan_waner/Realworld/calctopic1/derivgraph.html

Slope Fields

When the derivative is constant, it isn't too difficult to draw a graph of the function. However, when the derivative isn't constant (which happens more often), drawing an accurate picture is more difficult.

A **slope field** (or direction field) is a plot of short line segments indicating the slope of the function at a point for a lattice of points (x, y) in the plane. The slope field is defined by the equation of the derivative, $f\,'$,and can be used to determine the graph of the original function, f .

Example 5: Given the graph of $f\,'$, sketch the graph of the function f on the same set of axes if you know that $f(0) = 2$. Why is it necessary to know this last part?