3.1 DERIVATIVE OF A FUNCTION

In the last chapter we used a limit to find the slope of a tangent line. Without knowing it, you were finding a derivative all along. A derivative of a function is one of the two main concepts from calculus. The other is called an integral, and we will not get to that until later. The only change from the limit definition we used before, is that we are going to treat the derivative as a function derived from *f*.

Definition of a Derivative

The **derivative** of a function *f* with respect to the variable *x* is the function f' whose value at *x* is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

provided the limit exists.

Anywhere that the derivative exists, we say that the function is *differentiable*.

Example: Other notation used to denote the derivative (we will use most of these).

Example: Use the definition of the derivative to find $f'(x)$.

a) $f(x) = 3x + 2$

b) $f(x) = x^3 + x^2$

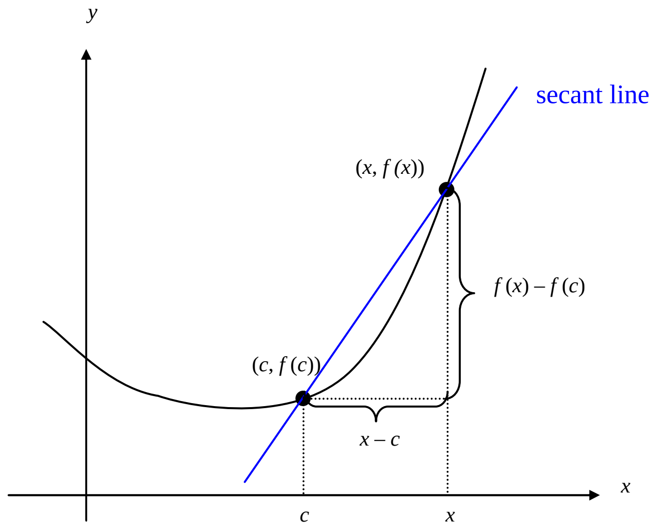
c) $f(x) = \frac{1}{\sqrt{x}}$

Alternative Definition #1 of the Derivative

An alternative definition of the derivative of *f* at *c* is

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

provided this limit exists.



♬: What this alternative definition allows us to do is to examine the behavior of a function as *x* approaches *c* from the left or the right. The limit exists (and thus the derivative) as long as the left and right limits exist and are equal.

Example: Use the alternative definition on the same examples as before.

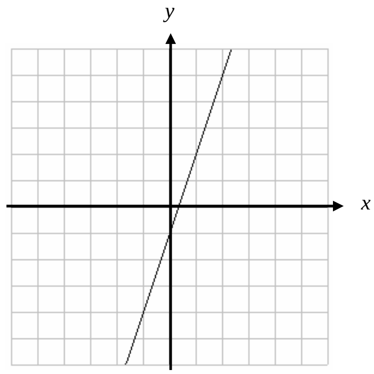
a) $f(x) = x^3 + x^2$

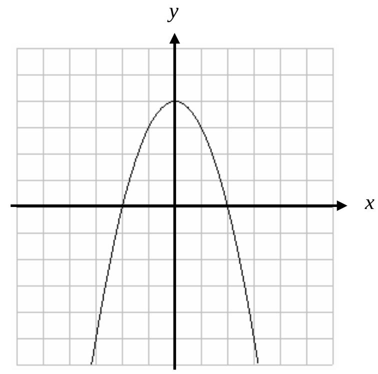
b) $f(x) = \dfrac{1}{\sqrt{x}}$

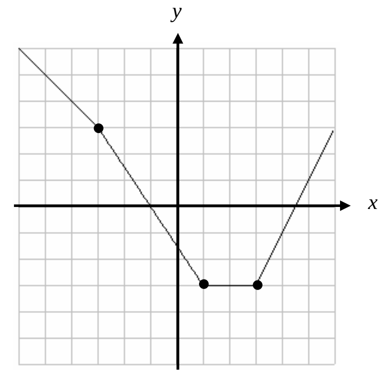
Relationships between the Graphs of f and $f\,'$.

Since a derivative at any point is equivalent to the slope of the function at that point, we can estimate what the original function looks like when we are given the graph of the derivative and vice – versa.

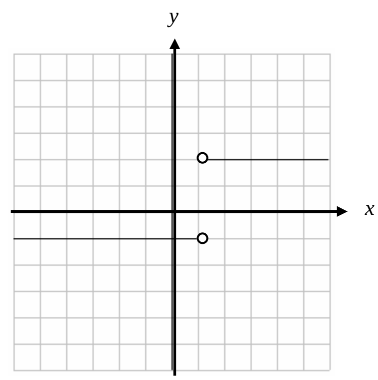
Example: Given the graph of *f*, sketch the graph of the derivative on the same set of axes.







Example: Given the graph of $f\,'$, sketch the graph of the function f on the same set of axes if you know that $f(0) = 2$. Why is it necessary to know this last part?



Slope Fields

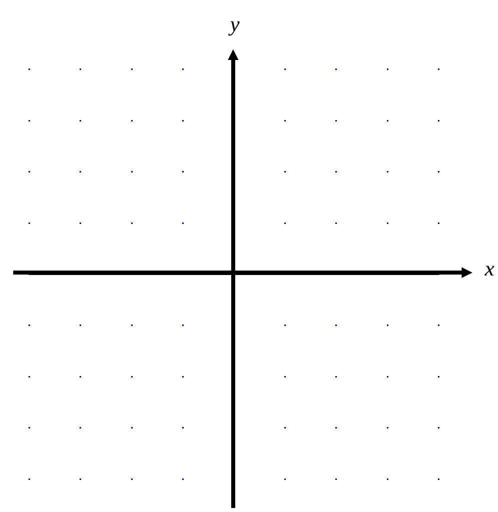
In the last example we were able to draw the graph of f when the derivative of f was constant. This obviously isn't the case most of the time. Slope fields enable us to draw a family of curves that have the same derivative.

> A **slope field** (or direction field) is a plot of short line segments indicating the slope of the function at a point for a lattice of points (x, y) in the plane. The slope field is defined by the equation of the derivative, $f\,'$,and can be used to determine the graph of the original function, f .

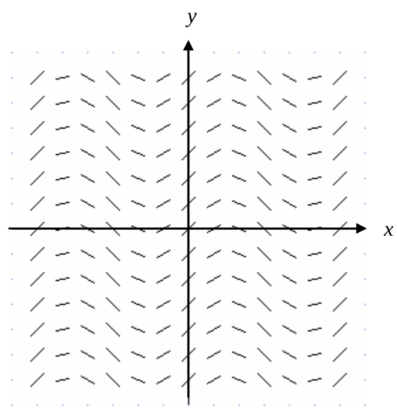
Example: Draw the slope field for $\dfrac{dy}{dx} = x$.

Example: Using the slope field draw 2 possible graphs for *y*.

Example: If the point (0, –3) is on the graph of *y*, draw the graph of *y*.



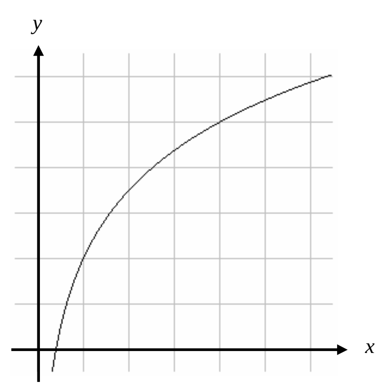
Example: Use the following slope field to sketch a possible graph of the function *f*.



Example: Sketch a function whose derivative is ALWAYS negative.

Example: Sketch a function whose derivative is ALWAYS positive.

Example: Identify or sketch each of the quantities on the figure to the right.



a) $f(1)$ and $f(4)$

b) $f(4) - f(1)$

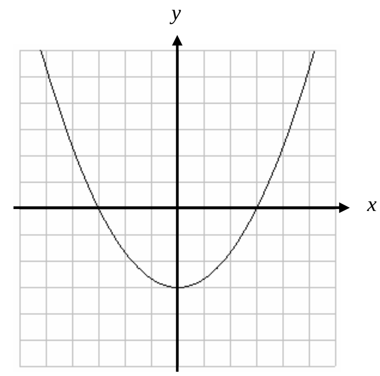
c) $y = \dfrac{f(4) - f(1)}{4-1}(x-1) + f(1)$

Example: Using the same graph, insert the proper inequality symbol (< or >) between the given quantities.

a) $\dfrac{f(4) - f(1)}{4-1}$ ☐ $\dfrac{f(4) - f(3)}{4-3}$

b) $\dfrac{f(4) - f(1)}{4-1}$ ☐ $f'(1)$

Example: The figure to the right shows the graph of g'.



a) $g'(0) =$

b) $g'(3) =$

c) What can you conclude about the graph of *g* knowing that $g'(1) = -\frac{8}{3}$?

d) What can you conclude about the graph of *g* knowing that $g'(4) = \frac{7}{3}$?

e) Is $g(6) - g(4)$ positive or negative? Explain.

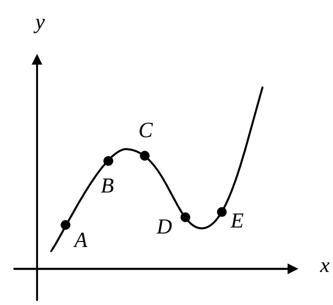
f) Is it possible to find *g* (2) from the graph? Explain.

Example: Assume that $f'(c) = 3$. Find $f'(-c)$ given the following conditions:

a) f is an odd function.

b) f is an even function.

Example: Use the graph of *f* at the right to answer each question.



a) Between which two consecutive points is the average rate of change of the function greatest?

b) Is the average rate of change of the function between *A* and *B* greater than or less than the instantaneous rate of change of *B*?

c) Sketch a tangent line to the graph between the points *B* and *C* such that the slope of the tangent line is the same as the average rate of change of the function between *B* and *C*.

d) Give any sets of consecutive points for which the average rates of change of the function are approximately equal.

Notecards from Section 3.1: Definition of a Derivative (2 of the 3 ways), Definition of the existence of a derivative at $x = c$ and at an endpoint, Slope Fields