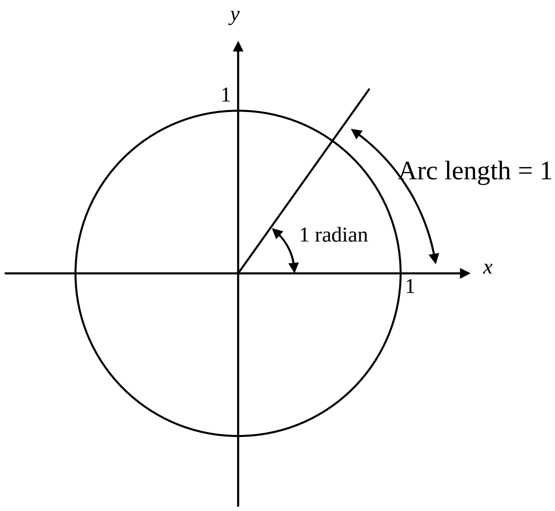
1.6 TRIGONOMETRIC FUNCTIONS

There are two common measures of angles: degrees and radians. As you'll see, almost all of calculus uses radians. Before beginning any exercise with trigonometric functions, make sure your calculator is set in *radian* mode. (ESPECIALLY THOSE YOU WHO HAD PHYSICS YESTERDAY!) Unless otherwise stated, the angles in the text are measured in radians. For example, sin 3 means the sine of 3 radians, but sin 3° means the sine of 3 degrees. Just for fun, you should understand exactly what a radian is.

Definition: Radian

An angle of **1 radian** is defined to be the angle at the center of a unit circle which spans an arc of length 1, measured counterclockwise.

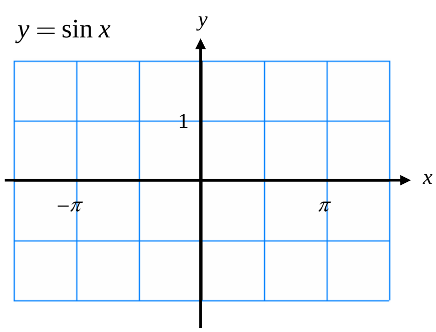


♬: You should also be VERY familiar with the 6 trigonometric values of the key points on the unit circle $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \ldots$

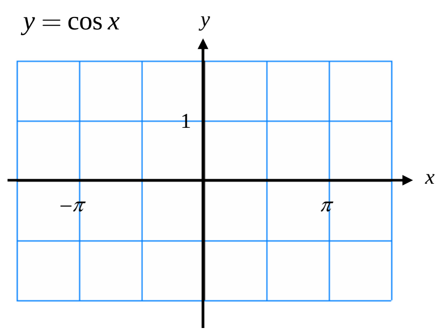
Graphs of Trigonometric Functions

Graph all 6 trigonometric functions below. Include both positive and negative values of *x*.

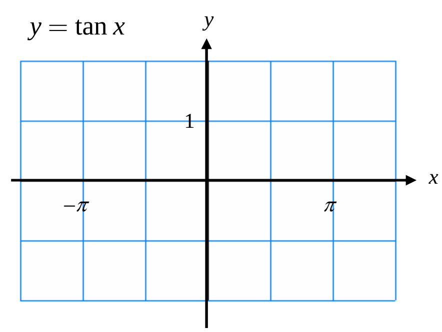
$y = \sin x$



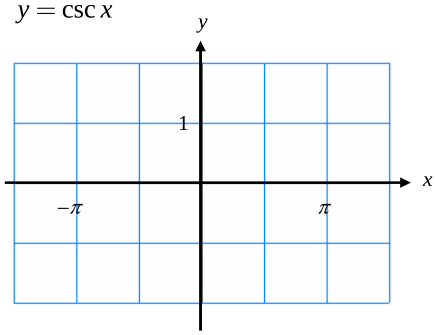
$y = \cos x$



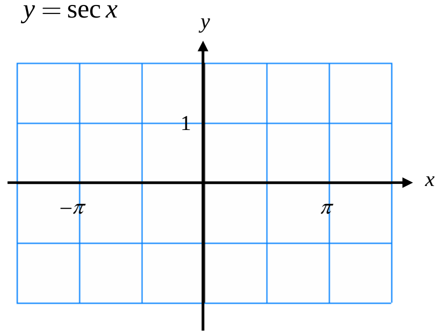
$y = \tan x$



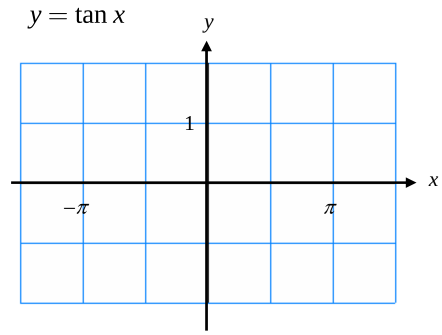
$y = \csc x$



$y = \sec x$



$y = \tan x$

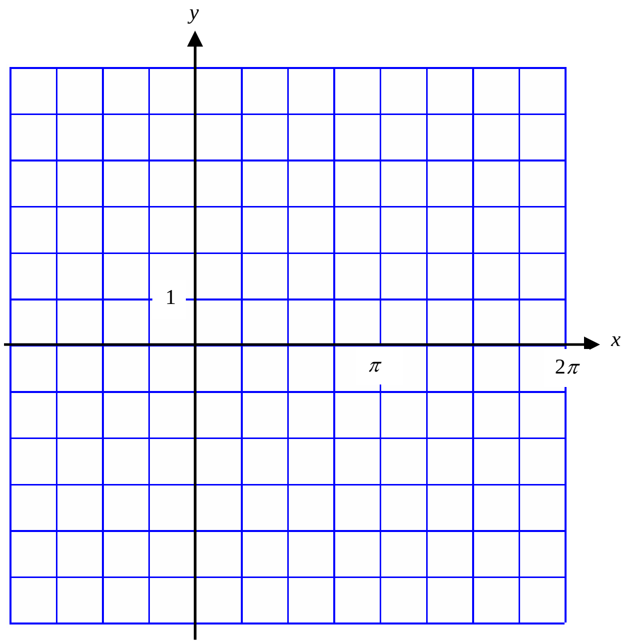


Transformations of Trigonometric Functions

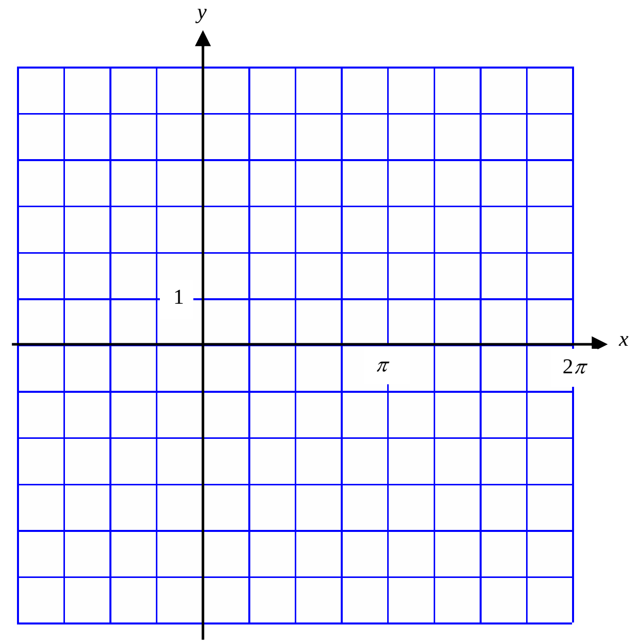
Example: For each of the following examples, do three things.

1. Describe the transformation.
2. Graph the function.
3. Where appropriate, give the amplitude and the period.

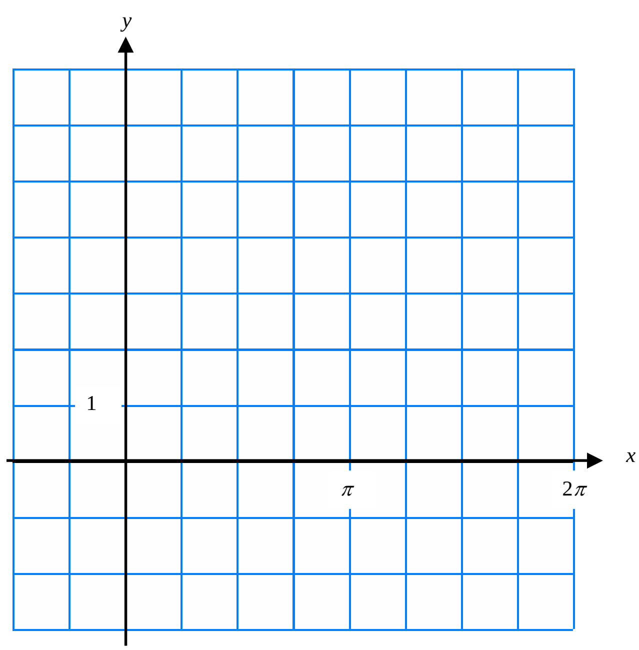
a) $y = 2\cos x - 3$



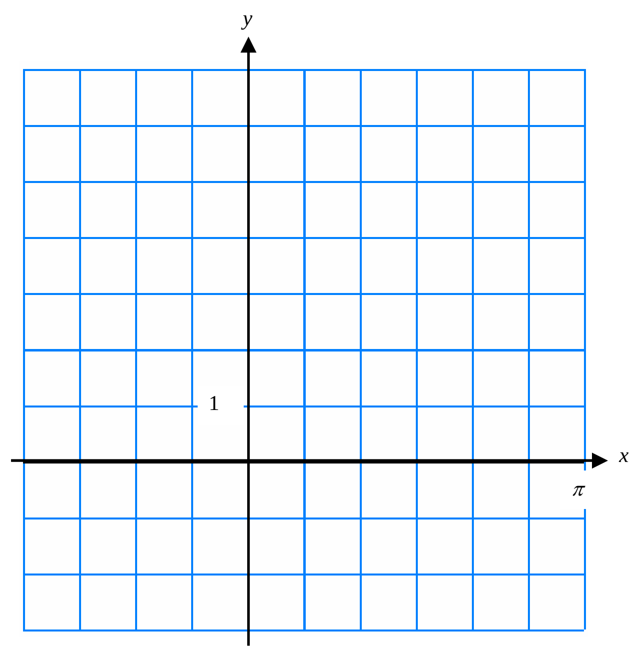
b) $y = \sin(-2x + \pi)$



c) $y = 2\sin(4x + \pi) + 3$



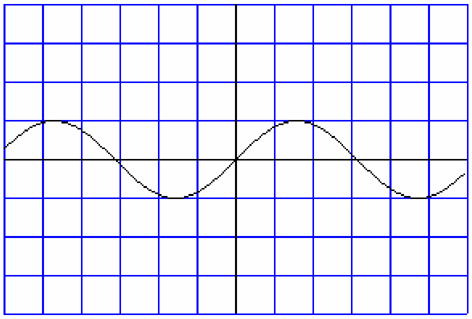
d) $y = -2\tan(3x - \pi) + 1$



Inverse Trigonometric Functions

Due to the periodic property of trigonometric functions, all 6 of the trig functions fail the horizontal line test (test for whether or not an inverse exists). In the graphs below, <u>color</u> (or highlight) the appropriate part of the trig function to use for its inverse, then <u>graph</u> it's inverse function.
Each box on each grid represents 1. You may wish to put appropriate values of π on the grid.

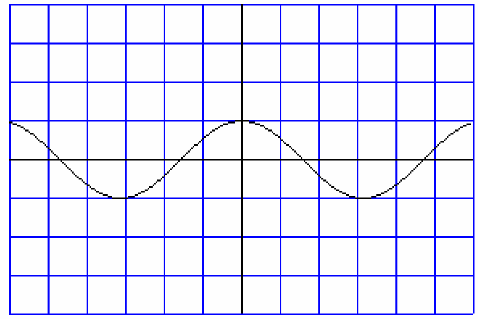
$y = \sin x$



Domain of Inverse:

Range of Inverse:

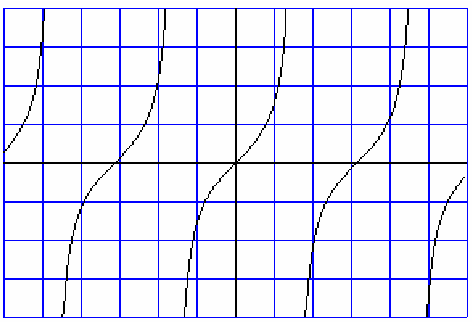
$y = \cos x$



Domain of Inverse:

Range of Inverse:

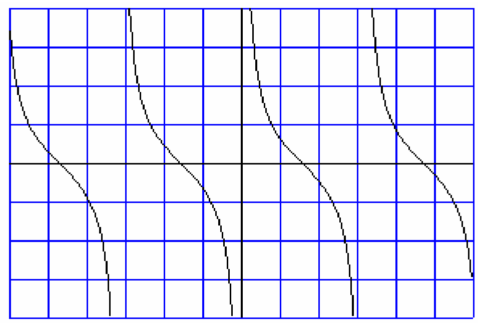
$y = \tan x$



Domain of Inverse:

Range of Inverse:

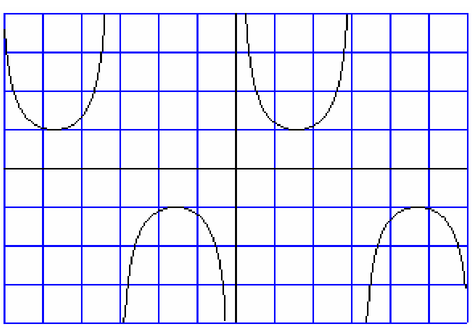
$y = \cot x$



Domain of Inverse:

Range of Inverse:

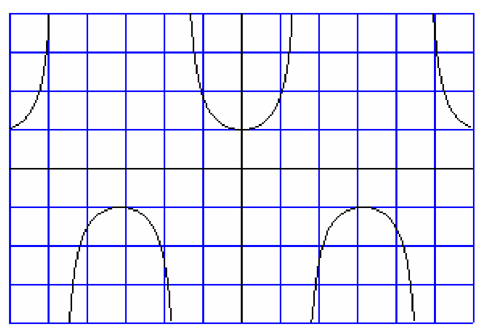
$y = \csc x$



Domain of Inverse:

Range of Inverse:

$y = \sec x$



Domain of Inverse:

Range of Inverse:

The restricted domain of the inverse trig functions means you must pay close attention to your solutions. Your calculator only gives you the solution for which the domain of the inverse function is defined. Your calculator also only has 3 of the six inverse functions.

Example: Find the domain and range of the following functions:

a) $\sin\left(\cos^{-1}(x)\right)$

b) $\sec\left(\tan^{-1}(x)\right)$

Example: Evaluate the expression WITHOUT a calculator. $\tan\left(\sin^{-1}\left(\frac{12}{13}\right)\right)$

Example: Solve for *x*: $\sec x = 3$ where $0 \le x \le 2\pi$.

The next two examples came from the 2007 AP Exam Free Response #4 (without a calculator). While the question itself focused on topics we will not cover until later in the year, the problems students had in answering the question stemmed from solving the following equations.

Example: Solve for t if $0 \le t \le 2\pi$. $e^{-t}\cos t + \sin t\left(-e^{-t}\right) = 0$

Example: Solve for A: $A\left(-2e^{-t}\cos t\right) + e^{-t}\left(\cos t - \sin t\right) + e^{-t}\sin t = 0$