1.5 FUNCTIONS AND LOGARITHMS

Notecards from Section 1.5: Finding and Proving Inverse Functions

Inverse Functions

In technical jargon, an inverse of a function maps the elements of the range to the elements of the domain. In English, this means that the inverse of a function reverses the domain and range. Not all graphs were defined as functions, and we had the *vertical line test* to determine whether a graph was or was not a function. Similarly, not all functions have an inverse that is a function, and we have the *horizontal line test* to determine whether or not a given function has an inverse function.

Definition: *One – to – One Function*

A function $f(x)$ is **one – to – one** on a domain D if $f(a) \neq f(b)$ whenever $a \neq b$.
A function that is one – to – one has an inverse.

The definition above can be seen graphically with the use of a horizontal line test. If there are two x – values for any given y – value of function, then the function does NOT have an inverse.

Example 1: Does $y = x^2 + 5x$ have an inverse? Why or why not?

Example 2: Does $y = x^3 + x$ have an inverse? Why or why not?

Once we know whether a function has an inverse, our next task is to find an equation and/or a graph for the inverse.

<u>Finding</u> *the Inverse Graphically (two ways)*

1. Reflect the graph of the original function over the line $y = x$.
2. Plot the reverse of the coordinates.

<u>Finding</u> *the Inverse Algebraically*

Switch the x and y in the original equation, then solve the new equation for y in order to write y as a function of x.

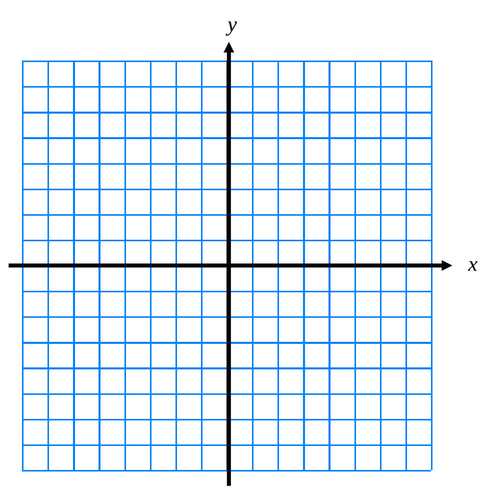
Example 3: Let $f(x) = x^3 - 1$.

a) Graph the function on the grid to the right.

b) Draw the line $y = x$

c) Reflect the graph of $f(x)$ over the line $y = x$.

d) ***Find*** the inverse of the function algebraically.



e) Use your graphing calculator to verify your answer to part *d*.

Verifying/Proving Inverses

It is one thing to ***find*** the inverse function (either graphically or algebraically), but it is another to ***verify*** that two functions are actually inverses. Whenever you are verifying anything in mathematics, you must go back and use the definition.

Definition: Inverse Function

A function $f(x)$ has an inverse $f^{-1}(x)$ if and only if $f\left(f^{-1}(x)\right) = x = f^{-1}\left(f(x)\right)$

Example 4: According to this definition, how many composite functions must you use to check whether or not two functions are inverses of each other?

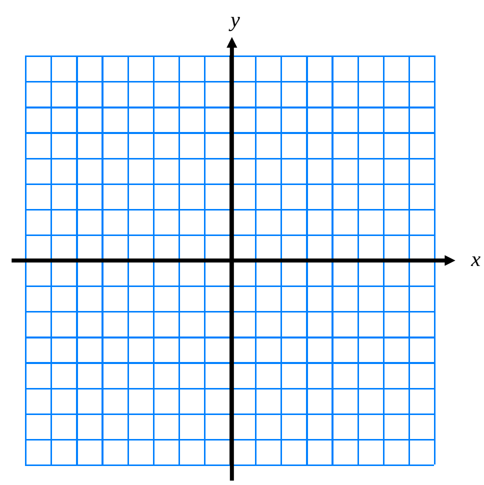
Example 5: Find $f^{-1}(x)$ and verify if $f(x) = \dfrac{x+3}{x-2}$.

Logarithmic Functions

How do Logarithms fit into this discussion? A logarithmic function is just the inverse of an exponential function.

Example 6: Graph $y = 2^x$ and find the inverse of the function graphically.

The equation of the inverse function is ________________.



Properties of Logarithms

Definition of a logarithm:

$$\log_a x = y \quad \Leftrightarrow \quad a^y = x$$

Inverse Properties of a logarithm:

$$a^{\log_a x} = x \qquad \log_a a^x = x$$

Logarithm of a product:

$$\log_a (xy) = \log_a x + \log_a y$$

Logarithm of a power:

$$\log_a \left(x^y\right) = y \cdot \log_a x$$

Logarithm of a quotient:

$$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$$

Change of Base Formula:

$$\log_a x = \frac{\ln x}{\ln a}$$

Other properties:

$$\log_a a = 1 \qquad \log_a 1 = 0$$

Example 7: Evaluate the following without using your calculator.

a) $\log_2 \frac{1}{8} =$

b) $\log_{27} 9 =$

Example 8: Solve for *x* in the following equations.

a) $\log_2 (x-1) = 5$

b) $3\left(5^{x-1}\right) = 86$

c) $\log(8x) - \log\left(1+\sqrt{x}\right) = 2$