1.5 FUNCTIONS AND LOGARITHMS

Inverse Functions

In technical jargon, an inverse of a function maps the elements of the range to the elements of the domain. In English, this means that the inverse of a function reverses the domain and range. Not all graphs were defined as functions, and we had the *vertical line test* to determine whether a graph was or was not a function. Similarly, not all functions have an inverse, and we have the *horizontal line test* to determine whether or not a function has an inverse.

Definition: *One – to – One Function*

A function $f(x)$ is **one – to – one** on a domain D if $f(a) \neq f(b)$ whenever $a \neq b$.
A function that is one – to – one has an inverse.

The definition above can be seen graphically with the use of a horizontal line test. If there are two x – values for any given y – value of function, then the function does NOT have an inverse.

Example: Does $y = x^2 + 5x$ have an inverse? Why or why not?

Example: Does $y = x^3 + x$ have an inverse? Why or why not?

Once we know whether a function has an inverse, our next task is to find an equation and/or a graph for the inverse.

Finding the Inverse Graphically (two ways)

1. Reflect the graph of the original function over the line $y = x$.
2. Plot the reverse of the coordinates.

Finding the Inverse Algebraically

Switch the x and y in the original equation, then solve the new equation for y in order to write y as a function of x.

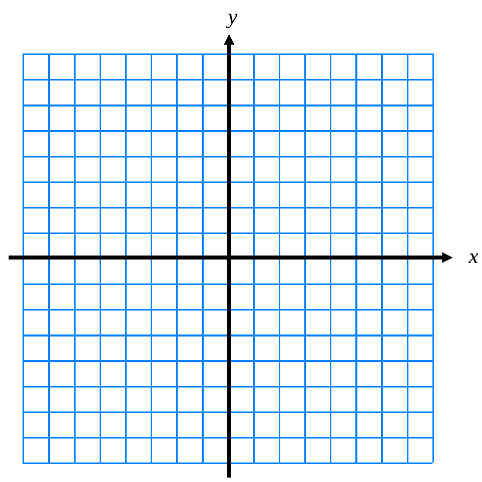
Example: Let $f(x) = x^3 - 1$.

a) Graph the function on the grid to the right.

b) Draw the line $y = x$

c) Reflect the graph of $f(x)$ over the line $y = x$.

d) Find the inverse of the function algebraically.



e) Use your graphing calculator to verify your answer to part *d*.

Verifying Inverses

It is one thing to find the inverse function (either graphically or algebraically), but it is another to verify that two functions are actually inverses. Whenever you are verifying anything in mathematics, you must go back and use the definition.

Definition: Inverse Function

A function $f(x)$ has an inverse $f^{-1}(x)$ if and only if $f\left(f^{-1}(x)\right) = x = f^{-1}\left(f(x)\right)$

Example: According to this definition, how many composite functions must you use to check whether or not two functions are inverses of each other?

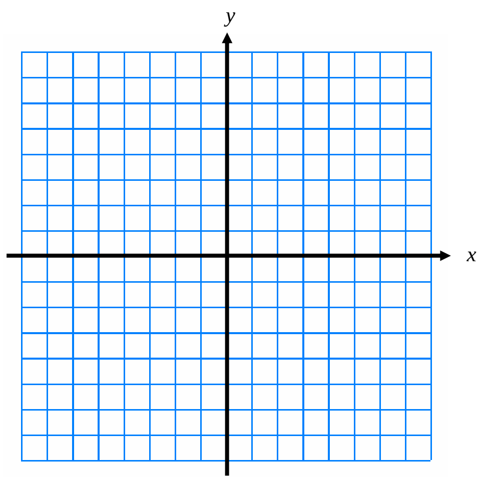
Example: Find $f^{-1}(x)$ and verify if $f(x) = \dfrac{x+3}{x-2}$.

Logarithmic Functions

How do Logarithms fit into this discussion? A logarithmic function is just the inverse of an exponential function.

Example: Graph $y = 2^x$ and find the inverse of the function graphically.

The equation of the inverse function is ________________.



Properties of Logarithms

Definition of a logarithm:

$$\log_a x = y \quad \Leftrightarrow \quad a^y = x$$

Inverse Properties of a logarithm:

$$a^{\log_a x} = x \qquad \log_a a^x = x$$

Logarithm of a product:

$$\log_a (xy) = \log_a x + \log_a y$$

Logarithm of a power:

$$\log_a \left(x^y\right) = y \cdot \log_a x$$

Logarithm of a quotient:

$$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$$

Change of Base Formula:

$$\log_a x = \frac{\ln x}{\ln a}$$

Other properties:

$$\log_a a = 1 \qquad \log_a 1 = 0$$

Example: Evaluate the following without using your calculator.

a) $\log_2 \frac{1}{8} =$

b) $\log_{27} 9 =$

Example: Solve for x in the following equations.

a) $3^{2x} = 75$

b) $3\left(5^{x-1}\right) = 86$

c) $e^{4x} = 15$

d) $\log_2 (x-1) = 5$

e) $\ln \sqrt{x+2} = 1$

f) $\log(8x) - \log\left(1+\sqrt{x}\right) = 2$

Example: The population of Glenbrook is 375,000 and is increasing at the rate of 2.25% per year. Predict when the population will be 1 million.

Example: Using the properties of logarithms, write y as a function of x.

$$\ln(y-1)-\ln 2 = x+\ln x$$

Example: Solve for x: $2^x + 2^{-x} = 5$

Example: The table below shows the amount of Canadian Oil Production since 1960.

Year	**Metric Tons (millions)**
1960	27.48
1970	69.95
1990	92.24

a) Find a natural logarithm regression equation for the data in the table.

b) Estimate the number of metric tons of oil produced by Canada in 1985.

c) Predict when Canadian oil production will reach 120 metric tons.

Notecards from Section 1.5: Inverse Functions