1.4 PARAMETRIC EQUATIONS

Notecards from Section 1.4: Trigonometric Identities

Up to this point all the functions we have been looking at have used a single equation with two variables, x and y. In this section we use a third variable to represent the curve. This third variable is called a parameter.

Example 1: Using your graphing calculator, set your window to $X_{min} = -5$, $X_{max} = 80$, $X_{scl} = 5$,
$$Y_{min} = -5, \quad Y_{max} = 20, \quad Y_{scl} = 5.$$

Graph $y = -\dfrac{x^2}{72} + x$, which models the path of an object thrown into the air at a 45° angle at an initial velocity of 48 feet per second. (Just take my word for it ☺)

Use TRACE to locate a few points on the graph. What do you learn about the object from this information?

Example 2: Now change your calculator to *parametric* mode and plot $x_1 = 24\sqrt{2}t$.
$$y_1 = -16t^2 + 24\sqrt{2}t$$

Notice it is the same graph … more on that later.
Use TRACE to locate a few points on the graph. What do you learn about the object from this information that you did not know before?

Definition: *Parametric Curve*

If x and y are given as functions
$$x = f(t), \quad y = g(t)$$
over an interval of t values, then the set of points $(x, y) = (f(t), g(t))$ defined by these equations is a parametric curve.
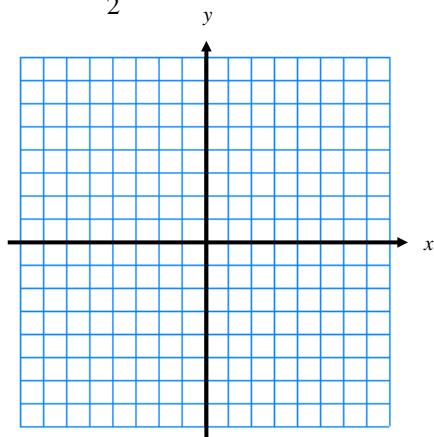

Graphing Parametric Curves Without a Calculator

Just like when you learned to graph for the first time back in Algebra 1, we are going to make a table of values. The difference is that we now have three variables instead of two.

Example 3: Graph the parametric curve
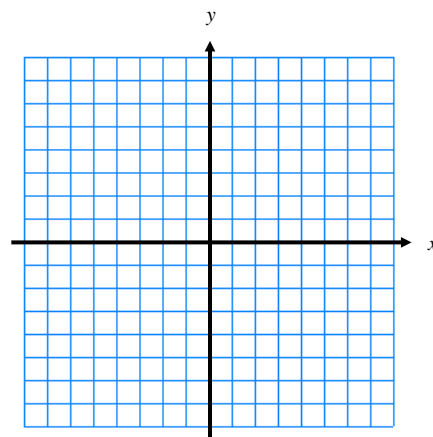$$x = t^2 - 4 \qquad -2 \le t \le 3$$
$$y = \frac{t}{2}$$

Example 4: Graph the parametric curve
$$x = 4t^2 - 4 \qquad -1 \le t \le \tfrac{3}{2}$$
$$y = t$$

*Example 5*: Compare and contrast the two graphs above.

Changing from Parametric to Rectangular (Cartesian)

To change a parametric equation back into a more familiar rectangular (Cartesian) equation you must eliminate the parameter. The typical approach to doing this is to solve for the parameter in one of the equations and then simply substitute that solution into the other equation.

Example 6: Change the parametric equation defined by $x = \sqrt{t}$ into a Cartesian equation.
$$y = 2 - t$$

The next two examples illustrate another way to eliminate the parameter by using trigonometric identities.

Example 7: Change the parametric equation $x = 3\cos\theta + 2$ into a Cartesian equation and then graph .
$$y = 5\sin\theta - 1$$
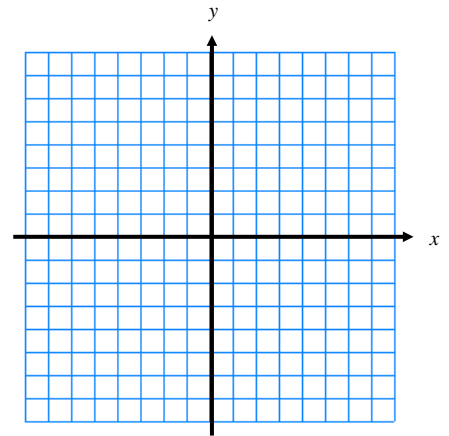
Example 8: Change the parametric equation defined by $x = 4\sec\theta$ into a Cartesian equation and then graph.
$$y = 3\tan\theta$$

Writing a Parametric Equation

Changing from Rectangular to Parametric means you get to create a parameter. As the graphs on the previous page indicated, your choice of parameter should not change the shape of the graph, only the "speed" in which the graph is drawn.

Example 9: Find a parametrization for the left half of the parabola $y = x^2 + 2x$.

Example 10: Find a parametrization for the line segment with endpoints (–1, 3) and (3, –2).