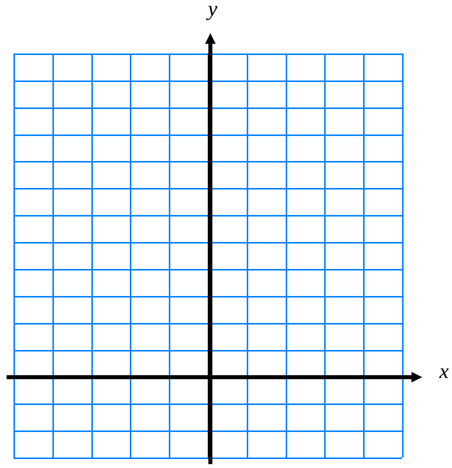
1.3 EXPONENTIAL FUNCTIONS

So far we've dealt with linear functions, piecewise functions, and composite functions. Next up, exponential functions.

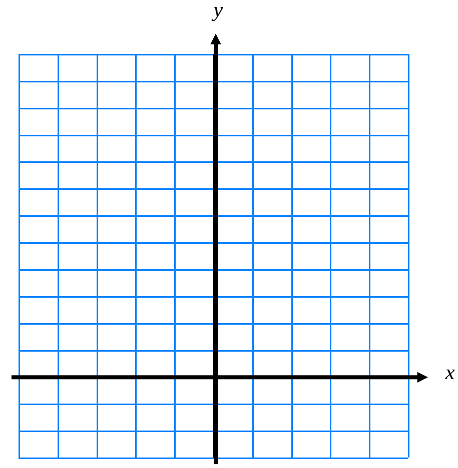
> *Definition*: *Exponential Function*
>
> If $b > 0$ and $b \neq 1$, then $f(x) = b^x$ is an exponential function with base b.

Example 1: Sketch the following graphs as accurately as possible on the graphs below:

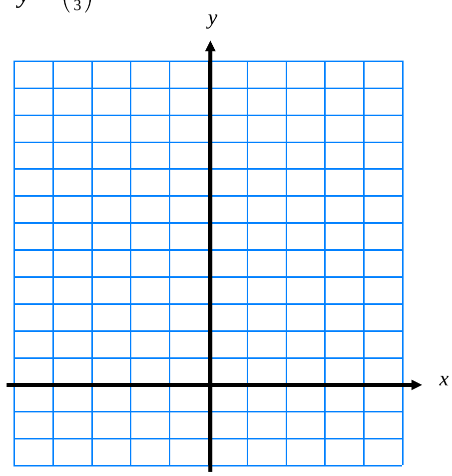
a) $y = 3^x$



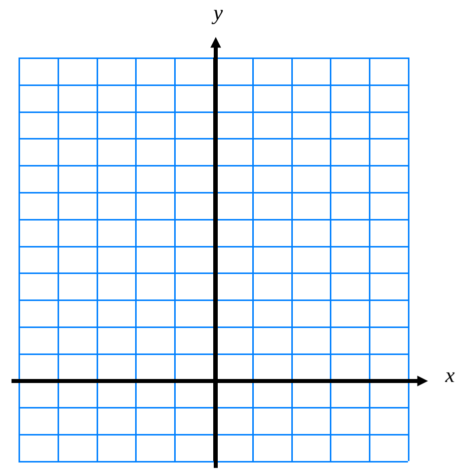
b) $y = 2 \cdot 3^x$



c) $y = \left(\frac{1}{3}\right)^x$



d) $y = 2 \cdot \left(\frac{1}{3}\right)^x$



Example 2: Which of these graphs show growth? decay?

Example 3: How does the 2 affect the graph?

Example 4: What is the domain and range of all 4 graphs?

Exponential Growth/Decay Model

In the exponential model $y = a \cdot b^x$, *b* is the rate of growth if *b* > 1, and *b* is the rate of decay if 0 < *b* < 1.
In either case, the initial value is *a*.

Example 5: Suppose you invest $12,000 in an account that earns you 5% interest compounded monthly for 10 years.

a) What is the initial amount?

b) What is the growth rate?

c) How many times does your money grow in 10 years? (How many times is interest added to your account?)

d) How much money will you have in 10 years?

The Number e

Many exponential functions in the real world (ones that grow/decay on a continuous basis) are modeled using the base of *e*.
Just like $\pi \approx 3.14$, we say $e \approx 2.718$. We can also define *e* using the function $\left(1+\frac{1}{x}\right)^x$ as follows:

$$\text{As } x \to \infty, \ \left(1+\tfrac{1}{x}\right)^x \to e$$

Try convincing yourself that this function approaches *e* using the [TABLE] function of your calculator.

Example 6: Suppose the interest in example 5 was compounded continuously. How much would you have in 10 years?

Example 7: Using your graphing calculator, let $Y_1 = x^2$ and $Y_2 = 2^x$. Graph both equations in the same window.

a) Solve the equation $x^2 = 2^x$ using your graphing calculator. Where are the solutions to this equation and how many are there?

b) Clear the two graphs from the screen and use the equation $x^2 - 2^x = 0$. Solve for *x* by graphing the left side of this equation. Where are the solutions to this equation and how many are there?

c) What did you learn from the last two questions?