1.1 Lines

Notecard: **Rules for Rounding** & **Average Rate of Change**
Round or Truncate all final answers to 3 decimal places. Do NOT round before you reach your final answer.

Goal Today: Write an equation of a line using a point and a slope.

Much of Calculus focuses on the concept of "local linearity", meaning that even if a function curves, if you were to pick a point and stay very close (local) to that point, the function behaves very much like that of a line.

Example 1: Graph the functions $y = \sin x$ and $y = x$ on your calculator. Obviously these are not the same function. However, if you were to stay close to the point (0, 0), these two functions are very close. Try zooming in on (0, 0) … try it more than once.

We can say that as long as we stay "close" to (0, 0), the functions $y = \sin x$ and $y = x$ are almost the same thing. Now, the concept of "close" is more complicated than it might sound, but more on that in chapter 2. For now, we focus on lines.

As stated in the syllabus, calculus has to do with change. For notational purposes, we use the capital Greek letter delta, Δ.

Slope of a Line … "AKA" ______________________________________

The slope of a non-vertical line is given by $\dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

A vertical line has ________________________, and a horizontal line has ________________________ .

Parallel Lines have slopes that are __________________ .

Perpendicular Lines have slopes that are ______________________________________.

Example 2: Selected values of the function $g(x)$ are given in the table. What is the average rate of change of $g(x)$ on the interval [–2, 5] ?

x	$g(x)$
–3	–60
–2	–18
1	0
4	108
5	220

Equations of a Line

The first equation of a line you used in algebra was probably the *slope – intercept form*: ___________________________
The slope is ______, and the y-intercept is ________.

It is actually easier to write the equation of a line in *point – slope form*: ____________________________
The point is ________, and the slope is __________.

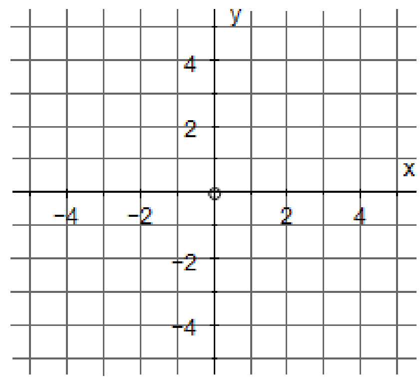
♬: To write an equation of a line, all you need is a ___________________ and the _________________.

Another format used to write the equation of a line is called *standard form*: _____________________________
All variables must be on the same side and A, B, and C are integers (no fractions or decimals!)

Which of the equations above has "y written as a function of x" ?

Example 3: The point-slope form is written as ______________________________ if you want "y written as a function of x"

Example 4: Suppose a line L has a slope of $-\frac{2}{3}$ and $L(-4) = 3$.



a) Write the equation of the line in point-slope form.

b) Use the equation of the line to find the value of $L(5)$.

c) Graph the line on the grid provided.

d) Write the equation of the line in standard form.

Example 5: Given a point $P(-2, 4)$ and a line L: $3x + 2y = 7$

a) Write an equation of a line through P parallel to L ______________________

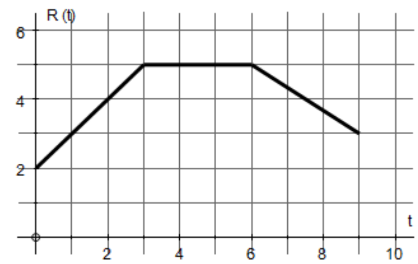
b) Write an equation of a line through P perpendicular to L ______________________

c) Write an equation of a line through P that is parallel to the x-axis. ______________________

Example 6: The graph of $R(t)$ shown below shows the rate in Liters per minute at which water is pumped into a tank during a 9 minute period.

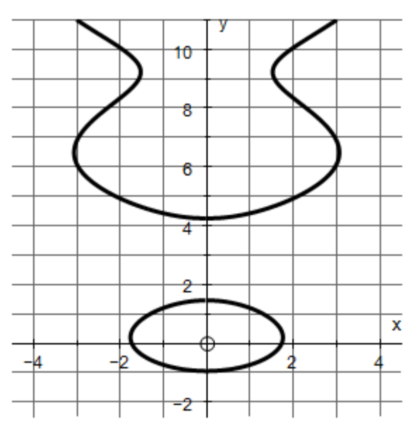
a) What is the average rate of change of $R(t)$ from $t = 2$ to $t = 5$? Indicate units of measure.

b) What is the average rate of change of $R(t)$ from $t = 1$ to $t = 8$? Indicate units of measure.



A little peak into your future … This was a <u>Calculator required</u> question from the 2015 AP Exam that I have reworded.

Example 7: The graph of $x^2 = -2 + y + 5\cos y$ is shown for values of $y \le 11$.



a) What is the y-value when $x = 2$. [*Hint:* It's not 5]

b) If you were told that the slope of this graph at any point (x, y) follows the formula $\dfrac{2x}{1-5\sin y}$, write the equation of the line when $x = 2$.

1.2 FUNCTIONS AND GRAPHS

Goal Today: Write and use piecewise and composite functions.

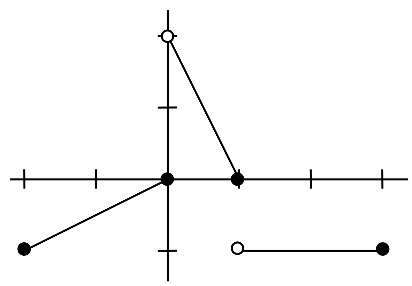
<u>*Question*</u>: If $y = \sqrt{x^2}$, what is the domain of this function? Given that domain, what does *y* equal?

Piecewise Functions

Piecewise functions are simply functions broken into "pieces". Each piece has its own domain.

Example 1: Sketch $f(x) = |x|$, and write an equation for the two "pieces" using a domain appropriate to each piece.

Example 2: Write a piecewise function for the graph at the right.



Example 3: Let $M(t) = \begin{cases} 5e^{-x} + 1 & \text{if } x < 1 \\ x^2 - 3x & \text{if } 1 \le x \le 3 \\ 5(x - 11) & \text{if } x > 3 \end{cases}$. Find the following:

a) $M(3) - M(7)$

b) The *y*-intercept of *M*.

c) a line perpendicular to $M(t)$ when $t = 5$.

Example 4: Suppose $f(x) = \begin{cases} 4 - x^2 & \text{if } x \le 3 \\ 2kx & \text{if } x > 3 \end{cases}$, where *k* is a real number. Find the value of *k* so that $f(x)$ is continuous.

Composite Functions

When the range of one function is used as the domain of a second function we call the entire function a composite function. In calculus, we will think of composite functions as having an "inside" and an "outside".

We use "circle" notation: $(f \circ g)(x) = f(g(x))$. This is read as "*f* composed with *g*" or "*f* of *g* of *x*".

In this notation, __________ is the "inside" function.

Example 5: If $f(x) = 1 - x^2$ and $g(x) = \sqrt{x}$, find $g(f(x))$. What is the domain and range of $g(f(x))$?

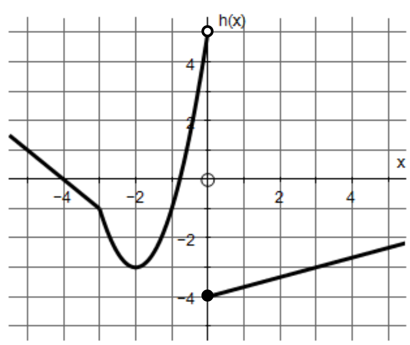
Example 6: If $k(x) = x^2 - 3x$, and $d(x) = x + h$ for some real number *h*, then what is $k(d(x))$?

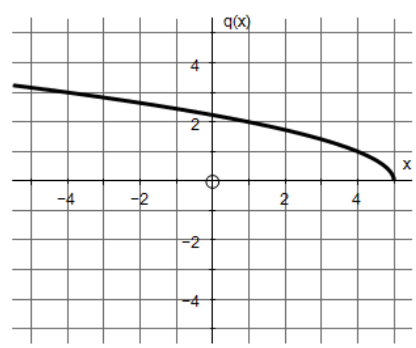
Example 7: If $f(x) = \dfrac{2x-1}{x+3}$, and $g(x) = \dfrac{3x+1}{2-x}$, find $f(g(x))$. Based on your answer, how might f and g be related?

Example 8: Let *h* (*x*) and *q* (*x*) be given.

a) What is $h(q(1))$?

b) What is the *y*-intercept of $q(h(x))$?





Example 9: Write the area of a circle in terms of the diameter using composite functions.

1.3 EXPONENTS AND LOGARITHMS

Goal Today: Simplify expressions and solve equations using logarithms and rational and negative exponents.

<u>*Question*</u>: Can you sketch the graphs of $y = e^x$ and $y = \ln x$?

Example 1: Without a calculator, what can you say about $g(x) = e^{-x^2}$ on the interval $0 < x < 2$?

a) g is always ________________ (positive or negative) on the interval.
b) g is always ________________ (increasing or decreasing) on the interval.

Exponent & Logarithm Rules … using e^x and ln(x)					
$e^a \cdot e^b =$	$\dfrac{e^a}{e^b} =$	$e^{-a} =$	$\dfrac{-7}{e^{-b}} =$	$e^{a/b} =$	$e^0 =$
$\ln a + \ln b =$	$\ln a - \ln b =$	$\ln a^b =$	$\ln 1 =$		
$e^{\ln a} =$	$e^{-2\ln a} =$	$\ln e =$	$\ln e^{5a-7} =$		

You will use these properties throughout the year to simplify expressions and solve equations and applications.

<u>*Simplify Expressions*</u>

Example 2: [MC] If $a = \log x$, which expression is equivalent to $\log \sqrt[5]{x^2}$?

A) $\frac{2}{5}a$ B) $\frac{5}{2}a$ C) $a^{2/5}$ D) $a^{5/2}$

Example 3: Without a calculator, evaluate the expression: $\frac{1}{2}\left[(3x-2)^2 - 4\right]^{-1/2} \cdot 2(3x-2)\cdot 3$ if $x = 3$.

Example 4: [MC] Simplify this expression: $5\left(\dfrac{x}{x+1}\right)^4 \left[\dfrac{(x+1)(1) - x(1)}{(x+1)^2}\right]$

A) $\dfrac{5x^4}{(x+1)^4}$ B) $\dfrac{-5x^5}{(x+1)^5}$ C) $\dfrac{5x^4}{(x+1)^6}$ D) $\dfrac{5x^4(2x+1)}{(x+1)^6}$

Example 5: Write the expression as a power function (in the form ax^n): $\dfrac{5}{7x\sqrt{x}}$

Solve Equations

Example 6: Solve for *x*: $\ln(x+2)-\ln x=3$

Example 7: Solve for *y* as a function of *t*: $\ln(y-1)-\ln(2)=t+\ln t$

Example 8: Rewrite the equation $\frac{G}{t^{4/5}}=-t^{3/4}(G-27)^{2/3}$ so that all the *G*'s are on one side and the *t*'s are on the other. Simplify, if possible. (*Write your answer so that other than the exponents, there are no fractions.*)

Solve Applications

Example 9: [No Calculator] Suppose $\frac{y^{-1}}{-1}=5x+C$, where *C* is a constant.

a) If and *y* (0) = 3, find the value of *C*.

b) Solve for *y* using the constant you found in part *a*.

Example 10: [Calculator Required] Bugs bunny had spent all day preparing for the prom. All the glitz and glamour of the evening fell apart when he tripped getting out of the limousine and fell flat on his face. Within minutes, news of Bugs' crash had begun spreading throughout the 525 people already at the prom. The function

$$P(t)=525\left(1-e^{-0.038t}\right),$$

where *t* represents the number of minutes after the fall, models the number of people who were already at the prom who have heard the news.

a) Find *P* (3.5) and explain its meaning using correct units in the context of this problem.

b) Find $\frac{P(4)-P(1)}{4-1}$ and explain its meaning using correct units in the context of this problem.

c) Solve the equation *P* (*t*) = 100 and explain its meaning in the context of this problem.

1.4 TRIGONOMETRY

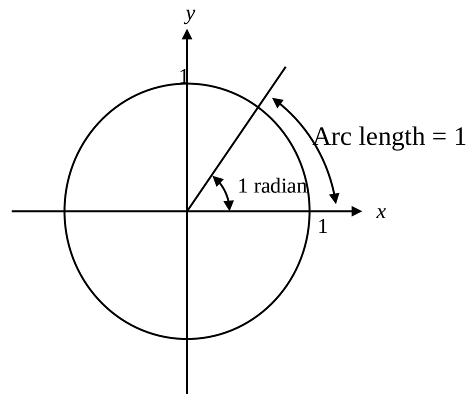
Goal Today: Use the unit circle and your calculator to simplify trig expressions.

As you saw in precalculus, there are two common measures of angles: degrees and radians. In this course, however, ***we always use radians***. Before beginning any exercise with trig functions, make sure your calculator is set in *radian* mode. (*Especially those of you who have Physics on your schedule!!*)

Definition: Radian

An angle of **1 radian** is defined to be the angle at the center of a unit circle which spans an arc of length 1, measured counterclockwise.

How many radians in an entire circle?



♬: You should also be VERY familiar with the 6 trigonometric values of the key points on the ***unit circle*** $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \ldots$

Example 1: Without a calculator, answer the following trigonometric expressions:

a) $\sin(\pi)$ b) $\tan\left(\frac{5\pi}{3}\right)$ c) $\cos(2\pi)$

d) $-\csc\left(\frac{\pi}{6}\right)\cot\left(\frac{\pi}{6}\right)$ e) $\sec^2\left(\frac{5\pi}{6}\right)$ f) $\csc(2\pi)$

g) $\tan^{-1}(1)$ h) $\sin^{-1}\left(\frac{1}{2}\right)$ i) $\cos^{-1}(-1)$

Example 2: What is the average rate of change on the graph of $y = \sin x$ on the interval $\left[\frac{\pi}{3}, \frac{3\pi}{2}\right]$?

Graphs of Trigonometric Functions

Example 3: You should be able to sketch each of the following functions and describe their period and amplitude.

$y = \sin x$ $y = \cos x$ $y = \tan x$

$y = \csc x$ $y = \sec x$ $y = \cot x$

The problems in the next example have been embedded in questions from previous AP exams.

Example 4: Solve each of the following without using your calculator

a) On the interval $0 \le x < 2\pi$, when does $\cos x = -\frac{1}{2}$?

b) For $t \ge 0$, find the first time that $2\cos(3t) = 0$

c) On the interval $0 \le x < 2\pi$, when does $e^{-x}\cos x + \sin x\left(-e^{-x}\right) = 0$?

d) Solve for A in the equation $A\left(-2e^{-t}\cos t\right) + e^{-t}\left(\cos t - \sin t\right) + e^{-t}\sin t = 0$.

Example 5: Use your calculator to solve the equation $-3 + \csc(-3\theta) = \dfrac{-9 + 2\sqrt{3}}{3}$ on the interval $-\pi \le x \le \pi$.

Example 6: When a particle has zero velocity, the particle is stopped. If the velocity of an object is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$, use your calculator to determine when the particle is stopped.

The next two examples deal with properties that become most important when we prove/explain other trig rules.

Example 7: Given that $\theta = \tan^{-1}\left(-\frac{5}{12}\right)$, find the value of sec θ.

Example 8: Without using a calculator, evaluate the following expressions:

a) $\sin\left(\cos^{-1}\left(\frac{7}{11}\right)\right)$

b) $\tan\left(\sin^{-1}\left(\frac{9}{13}\right)\right)$