### 3.7 OPTIMIZATION PROBLEMS

So far in Chapter 3, we have been studying the mathematical theory behind optimization. That is, finding the maximum or minimum values. Remember, the relative maximum or minimum values are found using the $\overline{\text { restricted, we must also test the }}$ $\qquad$ to find an absolute maximum or minimum.

In each of the problems in this section, your goal is to find the maximum or minimum value of a function representing some real-world quantity. Your first (and often the hardest) task is always to find an expression for the function to be optimized. This involves translating the problem into mathematical terms. Once you have the function, you can then apply the methods we have learned to determine the maximum or minimum value.

## Steps for Solving Applied Optimization Problems

Step 1: Understand the problem. Read it carefully, and ask yourself, "Self, what is the quantity to be maximized or minimized? What are the quantities which it depends on?"

Step 2: Draw a diagram if possible. Sometimes, more than one diagram helps you to determine how all the quantities are related. Identify all quantities from step 1 on your diagram.

Step 3: Assign variables to the quantities from step 1.
Step 4: Determine a function for the quantity to be optimized.
Step 5: Eliminate all but ONE variable and determine the domain of the resulting function. This usually requires a second equation.

Step 6: Optimize the function.
(a) Calculate the derivative and find the critical numbers
(b) If the domain is a closed interval, compare the function's value of the critical numbers with that of the endpoints.
(c) If the domain is an open interval (or infinite interval), use the first or second derivative tests to analyze the behavior of the function.

Example: Find two nonnegative real numbers that add up to 66 and such that their product is as large as possible.

Example: You are in a rowboat on Lake Erie, 2 miles from a straight shoreline taking your potential in-laws for a boat ride. Six miles down the shoreline from the nearest point on shore is an outhouse. You suddenly feel the need for its use. It is October, so the water is too cold to go in, and besides, your in-laws are already pretty unimpressed with your "yacht". It wouldn't help matters to jump over the side and relieve your distended bladder. Also, the shoreline is populated with lots of houses, all owned by people who already have restraining orders against you. If you can row at 2 mph and run at 6 mph (you can run faster when you don't have to keep your knees together), for what point along the shoreline should you aim in order to minimize the amount of time it will take you to get to the outhouse? (...And you thought calculus wasn't useful!)

Example: I don't know about you, but I wish that soda cans were bigger than 12 ounces. I'm thinking we need coke cans of 30 ounces (or about $887 \mathrm{~cm}^{3}$ ). However, you have been put in charge of designing this can without using too much material. What should the radius and height of the can be in order to minimize the amount of material used? Draw a picture of this shape...do you think this would go over with the marketing department?

