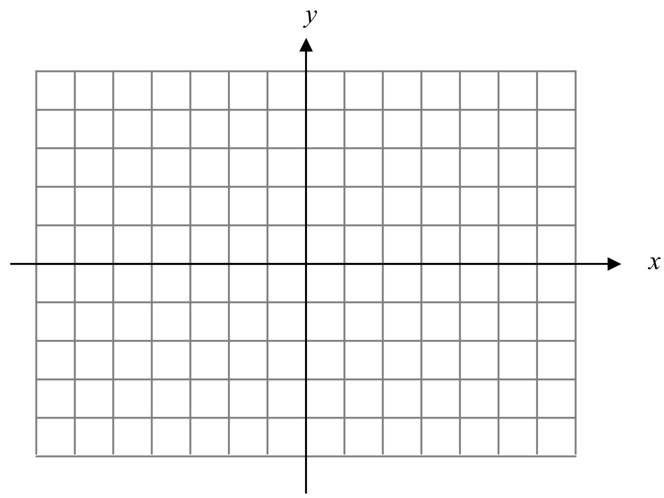
3.2 ROLLE'S THEOREM AND THE MEAN VALUE THEOREM

Rolle's Theorem

Example: Graph the points (–2, 3) and (5, 3). Draw a function that passes through both of these points.



Example: Is there at least one point on the graph for which the derivative is zero? Would it be possible to draw the graph so that there isn't a point for which the derivative is zero? Explain.

Theorem 3.3 Rolle's Theorem

Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b).

If $f(a) = f(b)$, then there is at least one number c in (a, b) such that $f'(c) = 0$.

Example: Determine whether Rolle's Theorem can be applied to f on the indicated interval. If Rolle's Theorem can be applied, find all values of c in the interval such that $f'(c) = 0$.

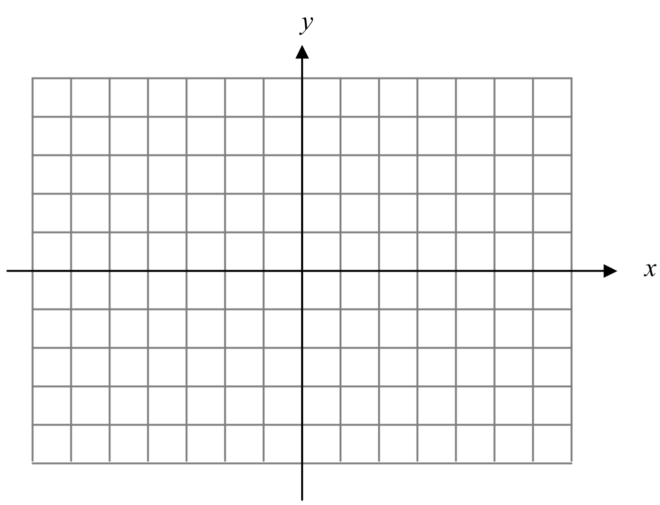
(a) $f(x) = x^2 - 3x + 2$ on the interval [1, 2]

(b) $f(x) = \dfrac{x^2 - 1}{x}$ on the interval [–1, 1].

The Mean Value Theorem

While there are applications for this next theorem, the most important contribution the theorem makes to the study of calculus is it’s usefulness in proving many of the theorems we use. Many people consider this to be one of the most important theorem in all of calculus. The proof of the Mean Value Theorem depends upon Rolle’s Theorem.

Example: Graph the points (–4, 6) and (5, –4). Draw a function that is continuous in the interval [–4, 6] and differentiable on the interval (–4, 6).



Example: Draw a line between the points (–4, 6) and (5, –4). Calculate the slope of this line.

Example: Are there any other points on your function, where the tangent line has the same slope as the line joining (–4, 6) and (5, –4) ? Sketch these tangent lines.

Theorem 3.4 The Mean Value Theorem

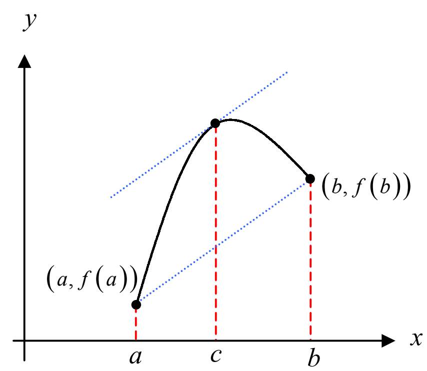
If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

Moreover,

$$f(b)=f'(c)(b-a)+f(a).$$

Proof:



Example: Apply the Mean Value Theorem to f on the indicated interval. In each case, find all values of c in the interval (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

(a) $f(x) = x(x^2 - x - 2)$ on the interval $[-1, 1]$

(b) $f(x) = \dfrac{x+1}{x}$ on the interval $\left[\dfrac{1}{2}, 2\right]$.