3.1 EXTREMA ON AN INTERVAL

Extrema of a Function

Calculus involves the study of functions and their behavior. A functions behavior is described by whether it is increasing or decreasing, what the rate of increase or decrease is, what the maximum and minimum values are, where the zeros of a function are, etc.

Definition of Extrema

Let f be defined on an interval I containing c.

1. $f(c)$ is the **minimum of *f* on *I*** if $f(c) \le f(x)$ for all x in I.
2. $f(c)$ is the **maximum of *f* on *I*** if $f(c) \ge f(x)$ for all x in I.

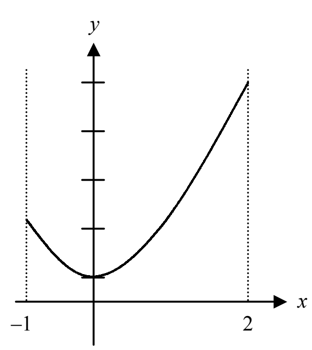
The minimum and maximum of a function on an interval are the extreme values or **extrema**.

Theorem 3.1 The Extreme Value Theorem

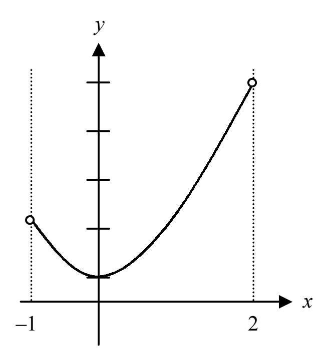
If f is continuous on a closed interval [a, b], then f has both a minimum and a maximum on the interval

Example: Using the graphs provided, find the minimum and maximum values on the given interval. If there is no maximum or minimum value, explain which part of the Extreme Value Theorem is not satisfied.

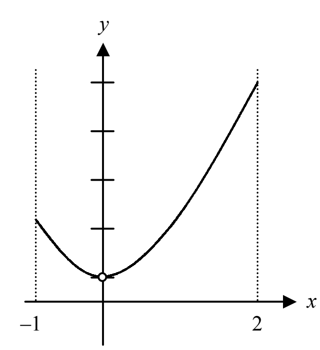
(a) [–1, 2]



(b) (–1, 2)

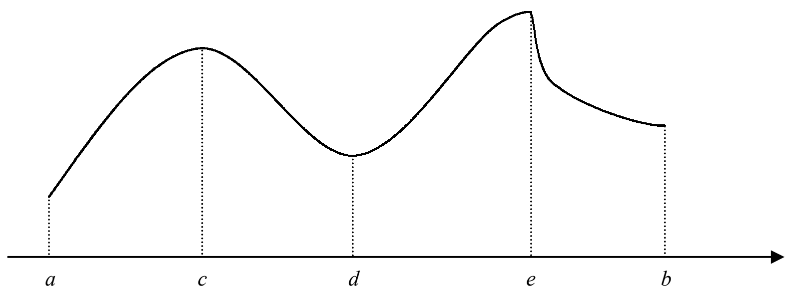


(c) [–1, 2]



Relative Extrema and Critical Numbers

Example: The maximum and minimum points in the last example occurred at either the endpoints or at points interior to the interval. Suppose our function looked like the graph below.



Definition of Relative Extrema

1. If there is an open interval containing c on which $f(c)$ is a maximum, then $f(c)$ is called a **relative maximum**.
2. If there is an open interval containing c on which $f(c)$ is a minimum, then $f(c)$ is called a **relative minimum**.

When given a graph it is simpler to identify the extrema. The question to be asked then is how do we find the extrema when we do not have a graph given to us?

Example: Except at the endpoints a and b, what do you notice about the derivative at the relative extrema in the last example?

Definition of a Critical Point

Let f be defined at c. If $f'(c) = 0$ <u>or</u> if f' is undefined at c, then c is a **critical point** of f.

Theorem 3.2 Relative Extrema Occur Only at Critical Points

If f has a relative minimum or relative maximum at $x = c$, then c is a critical number of f.

Since the r*elative extrema* can occur ONLY at critical points, and critical points occur ONLY when the derivative is either 0 or undefined, we can find the extrema on a *closed* interval using the guidelines below.

Guidelines for Finding Extrema on a Closed Interval

1. Find the critical numbers of f in (a, b).
2. Evaluate f at each critical number in (a, b).
3. Evaluate f at the endpoints of $[a, b]$.
4. The least of the values from steps 2 and 3 is the minimum, and the greatest of these values in the maximum.

Example: Find the extrema of $f(x) = x^{2/3}$ on the interval [–2, 3].

Example: Find the absolute extrema of the function $f(x) = \sqrt{4 - x^2}$ on the interval

(a) [–2, 2]

(b) [–2, 0)

(c) (–2, 2)

(d) [1, 2)

Example: Wile E. is after Road Runner again! This time he's got it figured out. Sitting on his ACME rocket he hides behind a hill anxiously awaiting the arrival that "beeping" bird. In his excitement to light the rocket he tips the rocket up. Instead of thrusting himself along the ground where he can catch the Road Runner, he sends himself into the air following a path given by the position function $s(t) = -16t^2 + 96t$. When does Wile E. Reach his maximum height?