Honors Algebra 2

2-3 Solving Quadratic Equations by Graphing and Factoring

Learning Targets

1. *I can solve quadratic equations by graphing*
2. *I can solve quadratic equations by factoring*
3. *I can write a quadratic function when given its roots*
4. *I can use the projectile model to determine the length of time the projectile is in the air*

SPECIAL FACTORING TECHNIQUES

Difference of Squares:

Perfect-Square Trinomials:

Additional Factoring Examples:

1) $9x^2 - 1$

2) $x^2 + 10x + 25$

3) $18x^2 - 48x + 32$

Zero: Any *x*-value that makes $f(x) = 0$.

FINDING ZEROS USING ALGEBRA

Zero Product Property: If (a)(b) = 0, then ____.

What is the converse of the Zero Product Property?

Examples: Find the zeros of each function by factoring:

4) $f(x) = 3x^2 + 18x$

5) $f(x) = 7x^2 + 12x + 5$

6) $f(x) = 4x^2 - 9$

7) $f(x) = 36x^2 - 84x + 49$

USING THE CONVERSE OF THE ZERO PRODUCT PROPERTY

For 8-10, write a function rule in factored form **AND** standard form with the given zeros:

8) 3 and –8

9) $\frac{3}{4}$ and 0

10) 7/3 and 2

FINDING ZEROS USING TECHNOLOGY

11) Finding the zeros of a function by using a graph or a table

a) $f(x) = x^2 - 6x + 2$

b) $f(x) = 3x^2 + 18x + 5$

How can you identify the zeros of a quadratic function from a table? From a graph?

Projectile Model: The height *h* in feet of an object (aka projectile) *t* seconds after it is launched into the air can be approximately modeled by the function $h(t) = -16t^2 + v_0 t + h_0$ where

$v_0 =$

$h_0 =$

12) A golf ball is hit from ground level with an initial velocity of 80 ft/s. After how many seconds will the ball hit the ground?

How do you know which values to use for v_0 and h_0 ?

How do you know which zero of the function to use as your answer?

Honors Algebra 2
2-4 Completing the Square

Warm-up:
Factor each expression

1. $3x^2 - 18x + 81$ 2. $16x^2 + 24x + 9$ 3. $9x^2 - 64$

Learning Targets

1. *I can solve quadratic equations using square roots*
2. *I can complete the square to write a quadratic function in vertex form*
3. *I can identify the vertex of a quadratic graph when given the function in vertex form*

Solve Equations using the square root property:

1. Solve:

a) $3x^2 + 5 = 41$ b) $\frac{1}{5}(x+3)^2 = 7$ c) $x^2 + 12x + 36 = 28$

Multiply.

2. a) $(x-4)^2$ b) $(x+3)^2$ c) $(x-7)^2$

Notice the relationship between "b" and "c" ……c is ___________________

Complete the square for each expression. Write the resulting expression as a binomial squared.

3. a) $x^2 + 20x +$ ______ $=$ ___________ b) $x^2 + 3x +$ ______ $=$ ___________

c) $x^2 - 8x +$ ______ $=$ ___________ d) $x^2 - 10x +$ ______ $=$ ___________

Notice the relationship between the number in (*x* - #), *"b" and "c" in the standard form ?*

NOTE: In all of the problems above, a = 1…., if a is not 1, you must factor it out 1st!!!!!

Solve each equation by completing the square

4. $2x^2 = 10 - 8x$

5. $0 = 5x^2 + 15x - 9$

6. $-3 = x^2 - 6x$

Review:

*Vertex Form:*___________________________

*Standard Form:*_________________________

What does "completing the square" do?

Write each function in vertex form, and identify its vertex.

7. $y = 2x^2 - 4x - 14$

8. $y = 5x^2 + 10x + 7$

Honors Algebra 2
2-5 Complex Numbers and Roots

Warm-up: NO Calculator

Simplify

1. $\sqrt{108}$

2. $\sqrt{6}\cdot\sqrt{24}$

3. $\dfrac{\sqrt{42}}{-\sqrt{3}}$

Find the zeros of the function without graphing:

4. $f(x)=x^2+8x-24$

Learning Targets

1. *I can use imaginary numbers to rewrite the square root of a negative number in terms of i.*
2. *I can use square roots to solve quadratic equations in terms of i.*
3. *I can determine a complex conjugate of a given complex number.*

Imaginary Number: is the square root of a negative number.

1. Express each number in terms of *i*.

   a) $5\sqrt{-121}$

   b) $-\sqrt{-96}$

   c) $\dfrac{1}{3}\sqrt{-63}$

2. Solve each equation

   a) $9x^2+25=0$

   b) $5x^2+90=0$

Complex Numbers: any number that can be written in the form *a* + *bi* where…

3. Find the values of x and y that make each equation true.

   a) $-8+(6y)i=5x-i\sqrt{6}$

   b) $4x+10i=2-(4y)i$

4. Find the zeros of each function

a) $f(x) = 3x^2 - 6x + 10$

b) $f(x) = x^2 + 16x + 73$

5. Solve each equation

a) $\frac{1}{2}x^2 + 72 = 0$

b) $x^2 + 3x + 14 = 0$

What do you notice about the solutions in the equations above?

Given that one solution of a quadratic equation is 3 + i, explain how to determine the other solution.

At which step in the solution process do you know that the function does not have real zeros? How do you know?

Complex Conjugates: For any complex number $a + bi$ form, the complex conjugate is $a - bi$.

6. Find the complex conjugate.

a) $8 - 5i$

b) $-9i$

c) 18

How does the complex number differ from its conjugate?

Honors Algebra 2
2.6 The Quadratic Formula

Learning Targets

1. *I can use the quadratic formula to solve quadratic equations with real roots.*
2. *I can use the quadratic formula to solve quadratic equations with complex roots.*

The derivation of the quadratic formula:

Solve for *x* by completing the square given $ax^2+bx+c=0$

The solution(s) for the quadratic equation $ax^2+bx+c=0 \quad (a \neq 0)$ is given by the **Quadratic Formula:**

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

YOU MUST MEMORIZE THIS!

Find the zeros of each function below by using the Quadratic Formula.

1. $f(x)=2x^2+8x-10$

2. $f(x)=4x^2+3x+2$

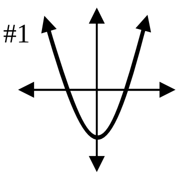
If a quadratic equation is in vertex form, what must you do before you can use the quadratic formula?

The Discriminant Theorem

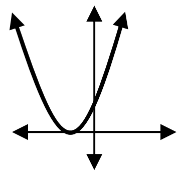
If a, b, and c are real numbers and $a \neq 0$, then the equation $ax^2 + bx + c = 0$ has (three possible scenarios):

1. two distinct real roots/solutions/zeros, if $b^2 - 4ac > 0$; two x-intercepts.
    - If $b^2 - 4ac$ is a perfect square the roots are also rational and the equation is factorable.
    - If $b^2 - 4ac$ is not a perfect square, the real roots are irrational.

#1

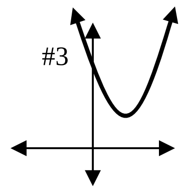


#2



2. one distinct real root/solutions/zeros, if $b^2 - 4ac = 0$; one x-intercept.
    - the root is also rational

#3



3. two distinct nonreal complex solutions, if $b^2 - 4ac < 0$; no x-intercepts.

Determine (a) the discriminant (b) the type and number of solutions for each equation and (c) how many x-intercepts the graph has.

3. $x^2 + 2x - 3 = 0$

(a)

(b)

(c)

4. $4x^2 - 12x = -9$

(a)

(b)

(c)

5. $4 = 6x - x^2$

(a)

(b)

(c)

6. An athlete on a track team throws a shot put. The height *h* of the shot put in feet *t* seconds after it is thrown is modeled by $h(t) = -16t^2 + 24.6t + 6.5$?

a) When does the shot put hit the ground?

b) When is the shot put 8 feet above the ground?

In preparation for the next lesson…

c) When is the shot put more than 8 feet above the ground? Less than 8 feet above the ground?

Honors Algebra 2
2.7 Solving Quadratic Inequalities

Warm-up

1. Give a solution to $y < 2x + 1$. Compare your solution with your classmates…how many solutions are there?

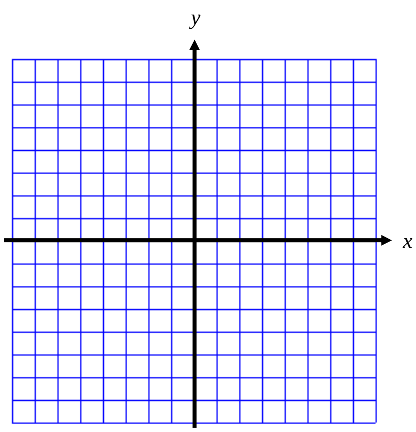
2. Graph the inequality: $y < 2x + 1$.
   Where are the solutions on the graph?

Learning Targets

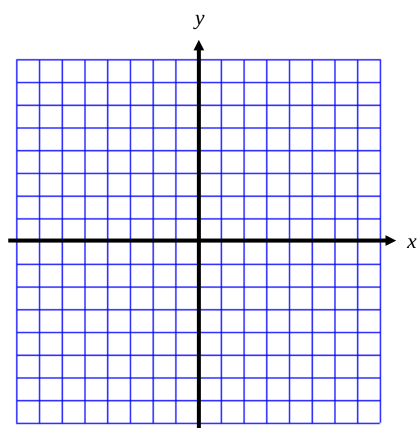
1. *I can graph a quadratic inequality.*
2. *I can solve a quadratic inequality using tables, graphs and/or algebra.*
3. *I can use the graph of a quadratic inequality to answer questions about real world situations.*

Graphing each Quadratic Inequality

1) $y \le x^2 - 8x + 12$

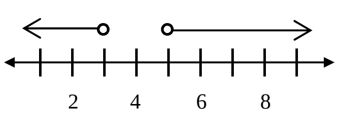


2) $y < -3x^2 - 6x - 7$

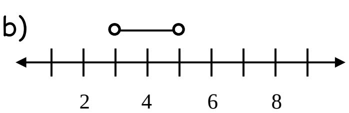


3. Write an inequality to describe each number line graph.

a)

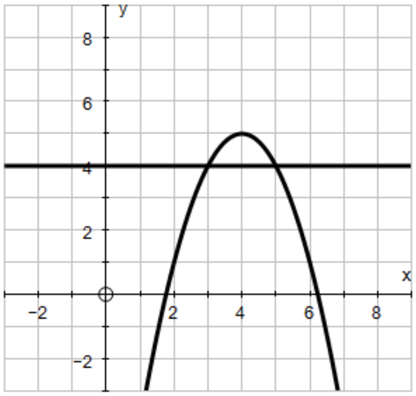


b)

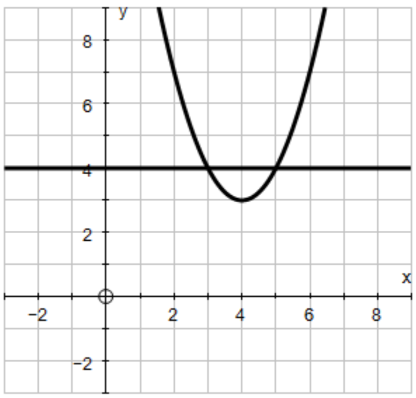


4. For what values of x is the parabola above the line? How do these solutions relate to question 3?

a)



b)



c) Given that the graph for part b comes from the inequality $x^2 - 8x + 19 > 4$, discuss the following…

- What part of the graph represents $x^2 - 8x + 19$?
- What part of the graph represents 4?
- Give an x value that makes the inequality true.
- Explain to your group WHY you selected that x value.

d) Solve $4 \le -x^2 + 8x - 11$ with a graphing calculator. How does this answer compare to 4a?

List the steps you would use to explain solving a quadratic inequality to a friend who was absent??

Solve the Quadratic Inequality with your graphing calculator.

5) $-10 \geq -x^2 + 6x - 15$

6) $x^2 - 6x - 72 < 0$

7) Answer the following questions in relation to #6…

a. How is the problem in question 6 different than the problems in questions 4 and 5?

b. What is another name for the intersection points you found in question 6?

c. Factor $x^2 - 6x - 72$. How would this help you solve question 6 without a graphing calculator?

d. Besides factoring (or using the quadratic formula), what else would you need to do to get the solution to $x^2 - 6x - 72 < 0$?

8) The monthly profit P of a small business that sells bicycle helmets can be modeled by the function $P(x) = -8x^2 + 600x - 4200$, where x is the average selling price of a helmet. What range of selling prices will generate a monthly profit of at least $6000? Use any method.

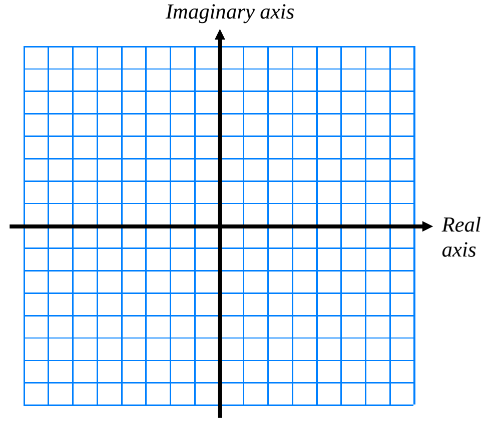
Honors Algebra 2

2-9 Operations with Complex Numbers

Learning Targets

1. *I can graph complex numbers.*
2. *I can add complex numbers.*
3. *I can subtract complex numbers.*
4. *I can multiply complex numbers*
5. *I can evaluate powers of* i.
6. *I can divide complex numbers.*

Complex Plane: The set of coordinate axes where the horizontal axis represents real numbers and the vertical axis represents imaginary numbers.



1. Graph each complex number.

a) $2 - 3i$ b) $-1 + 4i$

c) $4 + i$ d) $-i$

Absolute Value of a Complex Number

-

-

2. Find each absolute value.

a) $|3+5i|$ b) $|-5i|$ c) $|-4-8i|$

What does the absolute value of a complex number represent?

Adding and subtracting complex numbers: Combine the Real parts and the coefficients of the Imaginary parts.

3. Add or subtract. Write the result in the form *a + bi.*

a) $(4+2i)+(-6-7i)$ b) $-2i-(-2+3i)$ c) $(-30+i)-(-3+20i)$

Multiplying Complex Numbers: distribute or FOIL as needed. Remember to simplify using $i^2 = -1$.

4. Multiply. Write the result in the form *a* + *bi*.

   a) $-2i(2-4i)$ b) $(3+6i)(4-i)$ c) $(2+9i)(2-9i)$ d) $(-5i)(6i)$

Find the product of $(a+bi)(c+di)$*, and identify which terms in the product are real and which are imaginary.*

Dividing Complex numbers: Multiply by the complex conjugate of the denominator.

5. Simplify

   a) $\dfrac{3+10i}{5i}$

   b) $\dfrac{2+8i}{4-2i}$

Evaluating Powers of *i* *(See box at the bottom of page 128 and example 6 on page 129):*

6. Simplify

   a) $-6i^{14}$ b) i^{63}

How can you write an even power of i *as a power of* i^2 *?*

Honors Algebra 2
2-8 Curve Fitting with Quadratic Models

Warm-up:
Solve each system

1. $\begin{aligned} b &= -5 - 3a \\ 2a - 6b &= 30 \end{aligned}$

2. $\begin{aligned} 4a - 2b &= 8 \\ 2a - 5b &= 16 \end{aligned}$

Learning Targets

1. *I can use quadratic functions to model data.*
2. *I can use quadratic models to make predictions.*
3. *I can use quadratic models to analyze maximum and/or minimum values.*

Writing a Quadratic Function from Data

Example 1: Write a quadratic function that passes through (1, 4), (-2, 13) and (0, 3).

Do Exploration 2-8…Conclusions from Exploration:

Example 2: Determine whether the given set of data could represent a quadratic function. Explain.

a)

x	1	3	5	7	9
$f(x)$	-1	1	7	17	31

b)

x	3	4	5	6	7
$f(x)$	1	3	9	27	81

Now we know how to determine whether a list of data will be quadratic or not, and we can write quadratic functions when given data. Notice the system for example 1 was much easier to solve since one ordered pair had the *y* intercept in it. What if we don't have the *y* intercept or if the data is "messy"?

Quadratic Regressions

- To do a quadratic regression we need at least…

- To avoid rounding errors when we evaluate, we can…

Example 3: The table shows the cost of circular plastic wading pools based on the pools' diameter.

a) Find a quadratic model for the cost of the pool as a function of the diameter.

Diameter (ft)	Cost
4	\$19.95
5	\$20.25
6	\$25.00
7	\$34.95

b) Use the model to estimate the cost of a pool with a diameter of 8 feet.

c) What diameter pool will result in the minimum cost?