Honors Algebra 2
Chapter 4 Review

Name Cofield
Block

Non-calculator

1. Simplify using the properties of exponents. Leave answers as fractions where applicable. (Rev of Exp)

a) $(81^{\frac{5}{2}})^{\frac{1}{5}}$

$= 81^{\frac{1}{2}} = \boxed{9}$

b) $3^{\frac{1}{7}} \cdot 27^{\frac{-2}{14}}$

$= 3^{\frac{1}{7}} \cdot 27^{\frac{-1}{7}}$

$= 3^{\frac{1}{7}} \cdot (3^3)^{\frac{1}{7}}$

$3^{\frac{1}{7}} \cdot 3^{\frac{3}{7}} = \boxed{3^{4/7}}$

c). $\dfrac{36^{\frac{3}{4}}}{6^{\frac{1}{4}}} = \dfrac{(6^2)^{\frac{3}{4}}}{6^{\frac{1}{4}}}$

$= \dfrac{6^{\frac{6}{4}}}{6^{\frac{1}{4}}} = \boxed{6^{\frac{5}{4}}}$

d). $4^{\frac{-13}{4}} \cdot 4^{\frac{1}{5}} = 4^{\frac{-65}{20}} \cdot 4^{\frac{4}{20}}$

$= 4^{\frac{-61}{20}} = \boxed{\dfrac{1}{4^{61/20}}}$

e) $\left(\dfrac{1}{125}\right)^{\frac{2}{3}} = \left[\left(\dfrac{1}{125}\right)^{\frac{1}{3}}\right]^2$

$= \left(\dfrac{1}{5}\right)^2 = \boxed{\dfrac{1}{25}}$

f) $(4^{12})^{\frac{1}{3}} = 4^{\frac{12}{3}}$

$= 4^4$

$= \boxed{256}$

g) $16^{-\frac{3}{4}} = \left(16^{\frac{1}{4}}\right)^{-3}$

$= 2^{-3}$

$= \boxed{\dfrac{1}{8}}$

h) $\dfrac{a^{\frac{7}{3}}}{a^{\frac{5}{2}}} = \dfrac{a^{\frac{14}{6}}}{a^{\frac{15}{6}}}$

$= \boxed{\dfrac{1}{a^{1/6}}}$

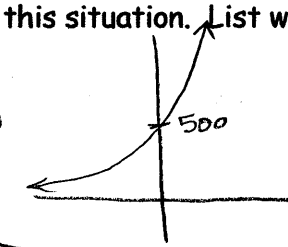
2. A population of 500 bacteria increases at a rate of 25% per minute. (4-1)

a. Write an equation and sketch a graph (by hand) to model this situation. List what each variable stands for.

P = population after m minutes

m = # of minutes

$\boxed{P = 500(1.25)^m}$



b. Tell whether this equation represents exponential growth, linear growth, or neither.

grows by a %

3. For each function: (4-2 parts a & b; 4-3 parts c & d)

i) Graph the function and its inverse in the same coordinate plane.

ii) Find an equation for $f^{-1}(x)$

a) $f(x) = -\dfrac{1}{2}x$

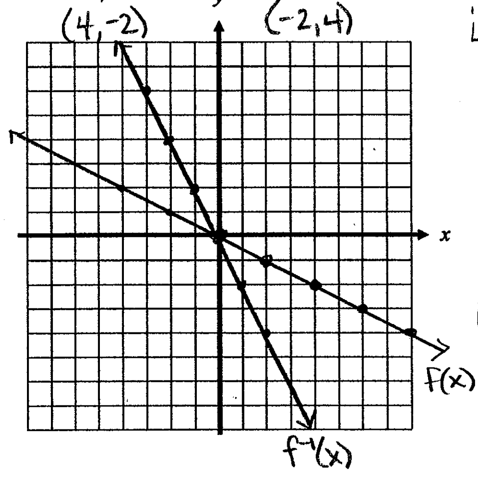
f(x)	$f^{-1}(x)$
(0,0)	(0,0)
(2,-1)	(-1,2)
(4,-2)	(-2,4)

ii) $y = \frac{-1}{2}x$

$x = \frac{-1}{2}y$

$-2x = y$

$\boxed{f^{-1}(x) = -2x}$



b) $f(x) = 2x - 5$

f(x)	$f^{-1}(x)$
(0,-5)	(-5,0)
(1,-3)	(-3,1)
(2,-1)	(-1,2)

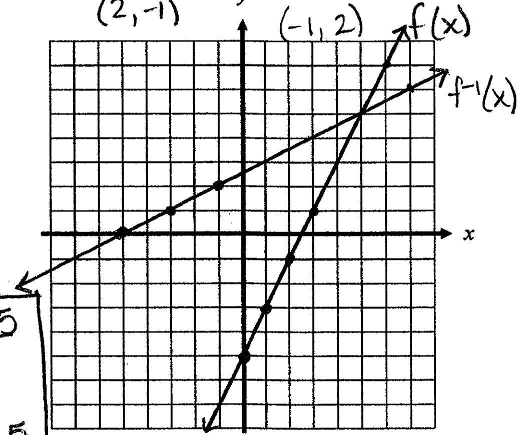
ii) $y = 2x - 5$

$x = 2y - 5$

$\dfrac{x+5}{2} = \dfrac{2y}{2}$

$\dfrac{x+5}{2} = y$

$\boxed{f^{-1}(x) = \dfrac{x+5}{2} \text{ or } f^{-1}(x) = \frac{1}{2}x + \frac{5}{2}}$



c) $f(x) = 3^x$

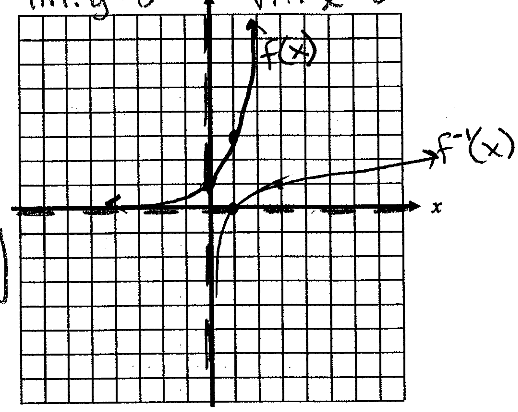
$f(x)$	$f^{-1}(x)$
$(0,1)$	$(1,0)$
$(1,3)$	$(3,1)$
HA: $y=0$	VA: $x=0$

ii) $y = 3^x$

$x = 3^y$

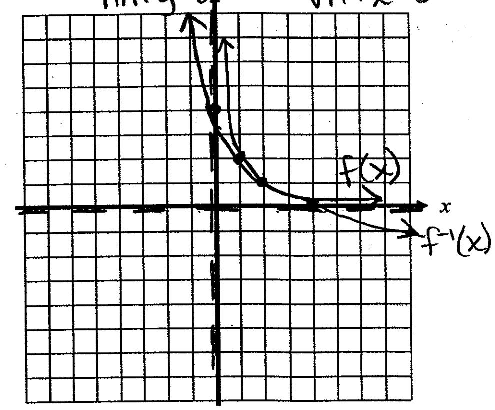
$\log_3 x = y$

$\boxed{f^{-1}(x) = \log_3 x}$



d) $f(x) = 4\left(\frac{1}{2}\right)^x$

$f(x)$	$f^{-1}(x)$
$(0,4)$	$(4,0)$
$(1,2)$	$(2,1)$
HA: $y=0$	VA: $x=0$



4. Write each equation in logarithmic form. (4-3)

a) $2^5 = 32$ $\log_2 32 = 5$

b) $4^{(-)\frac{1}{2}} = \frac{1}{2}$ $\log_4 \frac{1}{2} = -\frac{1}{2}$

c) $(0.5)^3 = 0.125$ $\log_{.5} 0.125 = 3$

5. Write each equation in exponential form. (4-3)

a) $\ln 148.41 \approx 5$

$e^5 = 148.41$

b) $\log_{\frac{1}{2}} 32 = -5$

$\left(\frac{1}{2}\right)^{-5} = 32$

c) $\log_{\frac{1}{25}} 125 = -\frac{3}{2}$

$\left(\frac{1}{25}\right)^{-3/2} = 125$

6. Evaluate each logarithm. (4-3)

a) $\log_6 36 = x$

$6^x = 36$

$6^x = 6^2$

$\boxed{x = 2}$

b) $\log_{27} 3 = x$

$27^x = 3$

$(3^3)^x = 3$

$3^{3x} = 3^1$

$3x = 1$ $\boxed{x = \frac{1}{3}}$

c) $\ln e^4 = \boxed{4}$

d) $\log 10^{0.1x} = \boxed{0.1x}$

d) $\log_4 1 = \boxed{0}$

e) $\log_{1.5} 2.25 = x$

$1.5^x = 2.25$

$\left(\frac{3}{2}\right)^x = \frac{9}{4}$

$\left(\frac{3}{2}\right)^x = \left(\frac{3}{2}\right)^2$

$\boxed{x = 2}$

f) $\log_{81} \frac{1}{3} = x$

$81^x = \frac{1}{3}$

$(3^4)^x = 3^{-1}$

$3^{4x} = 3^{-1}$

$4x = -1$

$\boxed{x = -\frac{1}{4}}$

g) $\ln e^{-2} = \boxed{-2}$

7. **Identify the parent function and describe how it was transformed into the given function. Determine the asymptote. Graph the function by hand including 2 key points and a dashed line for the asymptote. (4-7)**

a) $g(x) = 0.5(3)^x - 2$

parent: $f(x) = 3^x$ (0,1) (1,3) HA: y=0

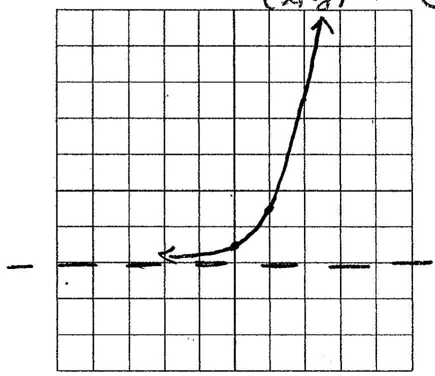
vert. compress $\frac{1}{2}$, down 2

$(x, y) \rightarrow (x, \frac{1}{2}y - 2)$

$(0, -\frac{3}{2})$

$(1, -\frac{1}{2})$

HA: y=-2



b) $h(x) = -e^{x-2}$

parent: $y = e^x$ (0,1) (1, 2.7) HA: y=0

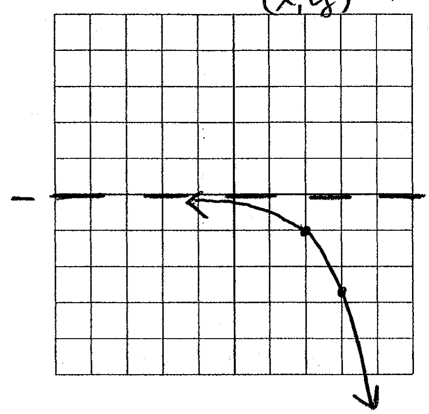
reflect over x-axis, right 2

$(x, y) \rightarrow (x+2, -y)$

(2,-1)

(3,-2.7)

HA: y=0



c) $g(x) = 5\log_3(x-2) - 3$

parent: $f(x) = \log_3 x$

(1,0)

(3,1)

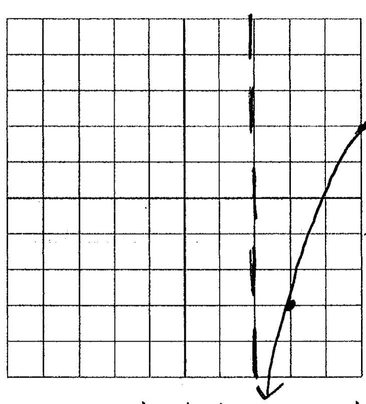
VA: x=0

$(x, y) \rightarrow (x+2, 5y-3)$

(3,-3)

(5, 2)

VA: x=2



vert. stretch 5, right 2, down 3

d) $h(x) = \ln(x+2)$

parent: $y = \ln x$

(1,0)

(2.7, 1)

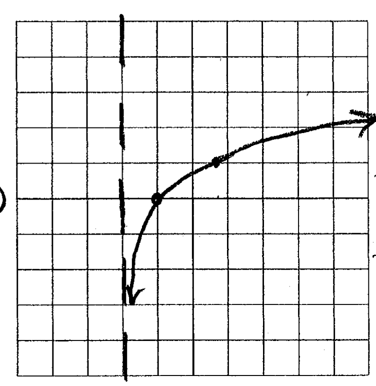
VA: x=0

$(x, y) \rightarrow (x-2, y)$

(-1,0)

(0.7,1)

VA: x=-2



left 2

8. **Rewrite the function given the transformation(s) below. (4-7)**

a) $f(x) = 4^x$ is reflected across the x-axis [redacted] and moved 3 units down and 4 units right.

$f(x) = -4^{(x-4)} - 3$

b) $f(x) = \ln x$ is compressed horizontally by a factor of $\frac{1}{2}$ and moved 2 units up.

$f(x) = \ln(2x) + 2$

c) $f(x) = 1.3^x$ is horizontally stretched by a factor of 1.5, reflected across the x-axis, and translated 1 units down.

stretch 3/2

$f(x) = -1.3^{(\frac{2}{3}x)} - 1$

d) $f(x) = \log x$ is translated 6 units right, vertically compressed by a factor of $\frac{1}{2}$ and translated 8 units up.

$f(x) = \frac{1}{2}\log(x-6) + 8$

Calculator Allowed:

9. Evaluate: (4-4 and 4-6) Use change of base or logBASE on calculator

a) $\log_{2.5} 30$ ← $\frac{\log 30}{\log 2.5} = \boxed{3.712}$

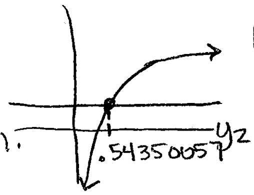
b) ln 3.78 $= \boxed{1.330}$

c) log 78 $= \boxed{1.892}$

10. Solve and check you answer. (4-5 and 4-6) You may solve log equations graphically(a) or algebraically(b)

a) $\underbrace{\log x}_{y_1} = \underbrace{-0.2648}_{y_2}$

Find intersection.



$\boxed{x = .544}$

b) $\ln x = 4.61$

$e^{\ln x} = e^{4.61}$

$x = e^{4.61}$

$\boxed{x = 100.484}$

c) $\frac{3e^{x-1}}{3} = \frac{4.8}{3}$

$e^{x-1} = 1.6$

$\ln e^{x-1} = \ln 1.6$

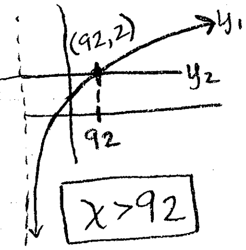
$x - 1 = \ln 1.6$

$x = \ln 1.6 + 1 = \boxed{1.470}$

11. Solve the following inequalities. Sketch a picture to explain your answer. (4-5 and 4-6)

a) $\underbrace{\log(x+8)}_{y_1} > \underbrace{2}_{y_2}$ $y_1 = \log(x+8)$ $y_2 = 2$

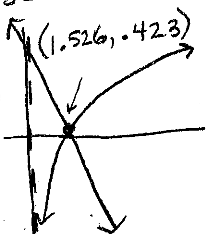
For what values of x is the logarithm above the line?



$\boxed{x > 92}$

b) $\ln(x) < 5 - 3x$ $y_1 = \ln x$ $y_2 = 5 - 3x$

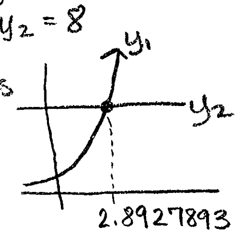
For what values of x is the natural log below the line?



$\boxed{0 < x < 1.526}$

c) $3^{x-1} > 8$ $y_1 = 3^{x-1}$ $y_2 = 8$

For what values of x is the exp. curve above the line?



$\boxed{x > 2.893}$

12. The table below shows the profits for several years, in thousands of dollars, of a company that produces computer software. ~~(4-1)~~ (4-8)

x = years after 1982	0	2	4	6	8	10	12
P = profits (thousand $)	452	761	1218	2067	3582	5205	8349

a) Find an exponential model for the data.

b) Use the model to estimate the profit in the year 1999.

(omit 2013)

13. If $7400 is deposited in an account at the bank and earns 11% annual interest, compounded continuously, what is the amount in the account, rounded to the nearest dollar, after 5 years? (4-6)

↳ use $A = Pe^{rt}$

$A = 7400e^{(.11 \cdot 5)}$

$\boxed{A = \$12826}$

* Notice $r = .11$ and not 1 + % like we use for $y = ab^x$

14. A mechanical engineer earned a yearly salary of $50,000 in 1990 and has averaged a 6.2% raise annually for the last 10 years and project that this increase will continue. (4-1)

$1 + .062 = 1.062$

a) Write an equation for the engineer's yearly salary, S, as a function of n, the number of years since 1990.

$\boxed{S = 50000(1.062)^n}$

b) Estimate the engineer's salary in 1980.

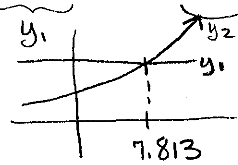
$n = -10$ $\quad S = 50000(1.062)^{-10}$

$\boxed{S = \$27398.38}$

c) How many years will it take for the salary to reach $80,000?

$\underbrace{\text{How many years}}_{n?}$

$\underbrace{80000}_{y_1} = \underbrace{50000(1.062)^n}_{y_2}$



(or) solve algebraically...

$\frac{80000}{50000} = \frac{50000(1.062)^n}{50000}$

$1.6 = 1.062^n$

$\log_{1.062}(1.6) = \log_{1.062}(1.062)^n$

$\log_{1.062} 1.6 = n$

$\boxed{7.813 \text{ yrs} = n}$

15. A headache medicine is eliminated from the bloodstream at a rate of 12% per hour. Suppose you take a 20 mg tablet at 4:00 pm.

$1 - .12 = .88$

a) Write an equation that models the amount of medicine in the bloodstream.

A = amount of medicine
h = # of hours after 4pm

$\boxed{A = 20(.88)^h}$

* Be sure to define your variables *

b) How many mg of medicine are left in the bloodstream at 5:30 pm?

5:30 pm = 1.5 hrs after 4pm

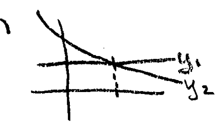
n?

$A = 20(.88)^{1.5}$

$\boxed{A = 16.510 \text{ mg}}$

c) How many hours will it take for half the medicine to be eliminated? Solve w/ graph or algebra.

$\underbrace{10}_{y_1} = \underbrace{20(.88)^n}_{y_2}$



(OR)

$A = 10$

$\frac{10}{20} = \frac{20(.88)^n}{20}$

$0.5 = .88^n$

$\log_{.88}(0.5) = \log_{.88}(.88)^n$

$\log_{.88}(0.5) = n$

$\boxed{5.422 \text{ hrs} = n}$

16. The radioactive isotope Germanium-71 has a half-life of about 15 days. (4-6)

a) Using $y = ae^{-kt}$, find the decay constant if a scientist has an initial amount of 25 grams.

$a = 25$

$y = \frac{25}{2} = 12.5$

$t = 15$

$\frac{12.5}{25} = \frac{25e^{(-k \cdot 15)}}{25}$

$0.5 = e^{-15k}$

$\ln 0.5 = \ln e^{-15k}$

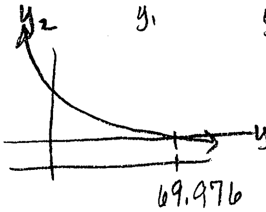
$\frac{\ln 0.5}{-15} = \frac{-15k}{-15}$

$\boxed{k = .046}$

b) Using your equation, how long until only 1 gram remains? Show/Explain your work!

$y = 25e^{-.046t}$ where t = days & y = amount left.

Graphically: $\underbrace{1}_{y_1} = \underbrace{25e^{-.046t}}_{y_2}$



When t ≈ 70 days, there is 1 gram remaining b/c that is when $y_1 = y_2$

(or) Algebraically

$1 = 25e^{-.046t}$

$\frac{1}{25} = e^{-.046t}$

$\ln 0.04 = \ln e^{-.046t}$

$\frac{\ln 0.04}{-.046} = \frac{-.046t}{-.046}$

$69.976 = t$

$\approx \boxed{70 \text{ days}}$

17. When calibrating a spring scale, you need to know how far the spring stretches for various weights. Hooke's law states that the length a spring stretches is proportional to the weight attached to it. A model for one scale is $l = 0.5w + 3$ where l is the total length (in inches) of the stretched spring and w is the weight (in pounds) of the object. (4-2)

a) Find the inverse of the given model. * Cannot switch variables for word problem!

$L = 0.5W + 3$

$L - 3 = 0.5W$

$2(L-3) = 2(0.5W)$

$\boxed{W = 2(L-3) \quad \text{or} \quad W = 2L-6}$

b) If you place a weight on the scale and the spring stretches to a total length of 6.5 inches, how heavy is the weight?

$W = 2L - 6$

$W = 2(6.5) - 6$

$W = 13 - 6$

$\boxed{W = 7 \text{ pounds}}$

18. The table below shows the growth in the number of radio stations after 1955. (4-8)

Find an exponential regression equation using your calculator. Round *a* and *b* to three decimal places. Use your model to predict the number of radio stations in 2001.

Years since 1955	0	5	10	15
Number of Radio Stations	3211	4133	5249	6760

omit 2013