Name KEY Block ______ Date ____________

Chapter 4 Review

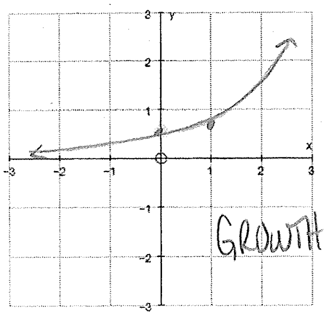
NON-CALCULATOR

On a separate piece of paper, thoroughly explain each of the following terms within the context of this chapter:

- asymptote
- base
- common logarithm
- exponential decay
- exponential equation
- exponential growth
- exponential regression
- inverse function
- inverse relation
- logarithm
- logarithmic equation
- logarithmic regression
- natural logarithm

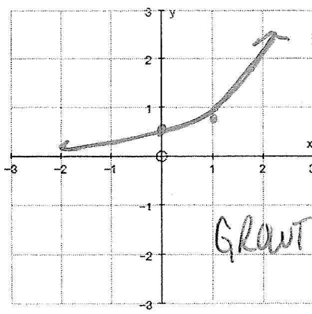
Tell whether the following function shows growth or decay. Then, sketch a graph of the function.

1. $f(x) = 0.5(1.25)^x$



x	y
0	.5
1	.625

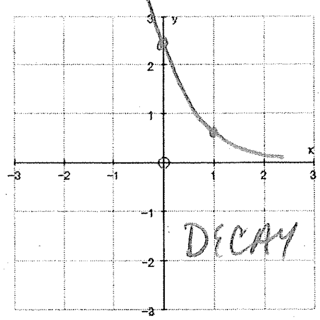
2. $f(x) = 0.5(\frac{3}{2})^x$



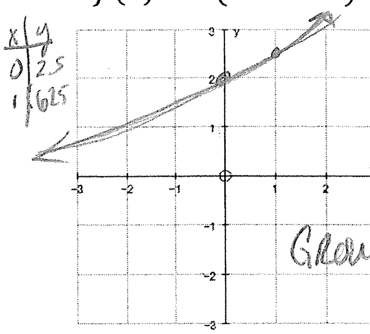
x	y
0	.5
1	.75

3. $f(x) = 2.5(0.25)^x$

x	y
0	2.5
1	.625



4. $f(x) = 2(1 + 0.25)^x$



x	y
0	2
1	2.5

5. Graph $f(x)=\frac{2}{3}x+2$. Then, find the inverse equation and graph it.

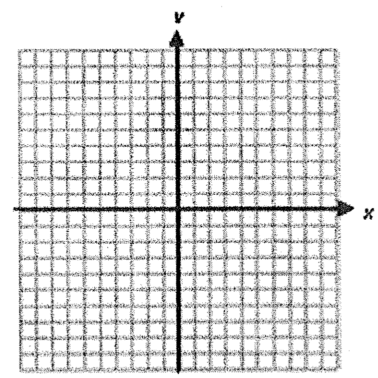
original: $y = \frac{2}{3}x + 2$

switch x & y: $x = \frac{2}{3}y + 2$

$(x-2)\frac{3}{2} = \left(\frac{2}{3}y\right)\frac{3}{2}$

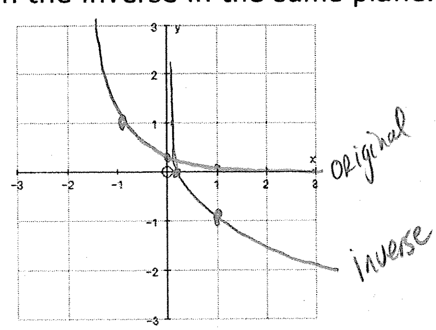
$\frac{3}{2}(x-2) = y$

$\boxed{\frac{3}{2}(x-2) = f^{-1}(x)}$



6. Graph the following relation and connect the points. Then graph the inverse in the same plane.

x	-1	0	1	2	3
y	1	0.2	0.04	0.008	0.001



7. The formula $M=\frac{5}{8}K$ gives the approximate distance in miles as a function of kilometers.

a) Find the inverse of this equation. In word PROBLEMS, Don't change the letters

$M = \frac{5}{8}K \Rightarrow \boxed{\frac{8}{5}M = K}$

b) Use the inverse equation to express 25 miles in kilometers.

$\frac{8}{5} \cdot 25 = \boxed{40 \text{ km}}$

Write each exponential equation in logarithmic form.

8. $3^5 = 243$ $\boxed{\log_3(243) = 5}$

9. $1 = 9^0$ $\boxed{\log_9(1) = 0}$

10. $\left(\frac{1}{3}\right)^{-3} = 27$ $\boxed{\log_{1/3}(27) = -3}$

Write each logarithmic equation in exponential form.

11. $\log_2 16 = 4$ $\boxed{2^4 = 16}$

12. $\log_{10} 10 = 1$ $\boxed{10^1 = 10}$

13. $2 = \log_{0.6} 0.36$ $\boxed{.6^2 = .36}$

Evaluate by using memorized exponents and mental math.

14. $\log_7 49 = x$

$7^x = 49$

$7^x = 7^2$

2

15. $\log_{0.5} 0.25 = x$

$.5^x = .25$

$.5^x = .5^2$

2

16. $\log_{12}\left(\frac{1}{12}\right) = x$

$12^x = \frac{1}{12}$

$12^x = 12^{-1}$

-1

17. $\log 0.01 = x$

$10^x = .01$

$10^x = 1/100$

$10^x = 10^{-2}$

-2

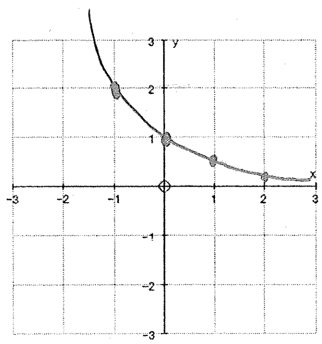
18. $\log_2 1 = x$

$2^x = 1$

0

19. Make a table of values for $f(x) = \left(\frac{1}{2}\right)^x$. Then, graph the function and its inverse. Finally, describe the domain and range of the ***inverse*** function.

x	-1	0	1	2
y	2	1	1/2	1/4



Domain: $\mathbb{R}$

Range: $y > 0$

Express each as a single logarithm and simplify.

20. $\log_2 8 + \log_2 16 =$

$3 + 4$

7

21. $\log_2 128 + \log_2 2$

$7 + 1$

8

22. $\log 100 + \log 10{,}000$

$2 + 4$

6

23. $\log 10 - \log 0.1$

$1 - (-1)$

2

24. $\log_5 25^2 = 2\log_5(25)$

$= 2 \cdot 2$

$= 4$

25. $\log 10^5 + \log 10^4$

$5 + 4$

9

26. $\ln e^{x+y} = x+y$

27. $e^{\ln(4x)} = 4x$

Solve and check.

28. $3^{x-1} = \frac{1}{9}$

$3^{x-1} = 3^{-2}$

$x - 1 = -2$

$\boxed{x = -1}$

29. $2^x = 4^{x+1}$

$2^x = 2^{2x+2}$

$x = 2x + 2$

$-x = 2$

$\boxed{x = -2}$

30. $\log_6(2x+3) = 3$

$6^3 = 2x + 3$

$216 = 2x + 3$

$213 = 2x$

$\boxed{\frac{213}{2} = x}$

31. $\log 50 + \log\left(\frac{x}{2}\right) = 2$

$\log(25x) = 2$

$10^2 = 25x$

$100 = 25x$

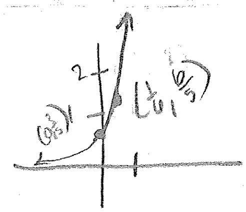
$\boxed{4 = x}$

32. Write the transformed function: $f(x)=e^x$ is reflected across the x-axis, stretched vertically by a factor of 3 and shifted 2 units down.

$g(x) = -3e^x - 2$

Give the intercept and asymptote for each function. Then, describe how the graph is transformed from the graph of the parent function.

33. $k(x)=\frac{3}{5}(2)^{6x}$



y-intercept: $(0, 3/5)$

asymptote: $y=0$

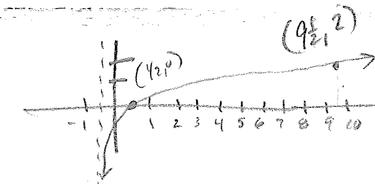
description: $y=2^x$ PARENT

vertical compression by a factor of 3/5

Horizontal compression by a factor of 1/6

$(x,y) \to (\frac{1}{6}x, \frac{3}{5}y)$

34. $m(x)=2\log\left(x+\frac{1}{2}\right)$



x-intercept: $(1/2, 0)$

asymptote: $x=-1/2$

description: $y=\log(x)$ Parent

vertical stretch by a factor of 2

Left 1/2 $\quad (x,y) \to (x-1/2, 2y)$

CALCULATOR

35. The student population in a small resort town has increased by 2% per year for the last 5 years. This year's population is 765 students.

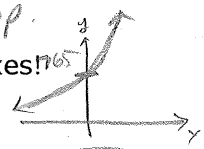
$b=1.02$

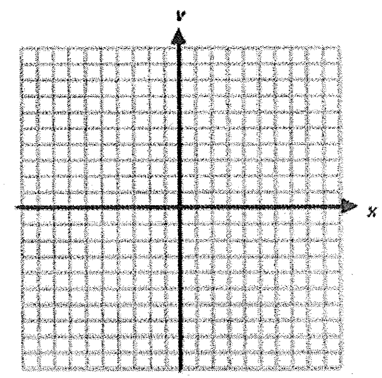
a) Will the function that represents this situation show growth or decay? GROWTH

b) Suppose that the student population continues to follow the same trend. Write a function to show the number of students as a function of the year, starting with the current year.

$y=765(1.02)^x$ $\quad$ x = yrs after this one $\quad$ y = pop.

c) Sketch a graph of the function. Be sure to label your axes!





d) Predict the number of students in 3 years.

$y=765(1.02)^3 \approx 811.82412$ ≈ 812

e) Now using your calculator, graph and predict when the population will exceed 1000 students.

$\underbrace{1000}_{y_1} = \underbrace{765(1.02)^x}_{y_2}$ $\quad$ calc. intersection $\quad \boxed{x \approx 13.527147}$

36. The apparent loudness of music today at Sam's Cafe was 10 decibels louder than it was yesterday. Apparent loudness L is given by $L=\log\frac{I}{I_0}$, where I is the intensity of the sound in W/m^2 and I_0 is the lowest intensity that the ear can detect. How many times more intense was the sound today than yesterday?

WANT $\frac{\text{intensity today}}{\text{intensity yesterday}}$

Solve $L=\log(I/I_0)$ for I

$10^L = I/I_0$

$I_0 \cdot 10^L = I$

if yesterday had loudness L decibels then today was L+10

$\frac{10^{L+10}\cdot \cancel{I_0}}{10^L \cdot \cancel{I_0}} = \boxed{10^{10} \text{ times intense}}$

Evaluate.

37. $\log_3 15 = \frac{\log(15)}{\log(3)} \approx \boxed{2.464973521}$

38. $\log_8 21 = \frac{\log(21)}{\log(8)} \approx \boxed{1.464105808}$

Solve.

39. $\underbrace{\log x^{5/2}}_{y_1} > \underbrace{2.5}_{y_2}$

$\boxed{x>10}$

40. $\underbrace{\left(\frac{1}{2}\right)^x}_{y_1} \le \underbrace{64}_{y_2}$

$\boxed{x \ge -6}$

41. $\ln x - 2 = 5$

$\ln x = 7$

$e^7 = x$

$x \approx 1096.633$

42. $2\ln x - \ln x = 3$

$\ln x = 3$

$e^3 = x$

$\boxed{x \approx 20.086}$

$P_0 = 22$ $t = 0$

43. The population of whooping cranes was about 22 in 1940 and grew at an exponential rate to about 194 in 2003. when $t = 63$ $P = 194$

a) Use the exponential growth function $P(t) = P_0 e^{kt}$, where P_0 is the initial population and $P(t)$ is the population at time t, to determine the growth factor, k.

$P = 22e^{kt}$ $\quad 194 = 22e^{63k}$ $\nearrow$ $\frac{194}{22} = e^{63k}$ $\to$ $\ln\left(\frac{194}{22}\right) = 63k$ $\nearrow$ $\frac{\ln(194/22)}{63} = k$ $\quad k \approx .0345526302$

b) If the flock continues to grow at the same rate, how large will it be in 2020?

$P = 22e^{k \cdot 80} \approx 349.0631576$ $\quad \boxed{\approx 349}$ $\quad t = 80$

c) When will the population reach 300?

$300 = 22e^{kt}$ $\quad \frac{300}{22} = e^{kt}$ $\to$ $\ln\left(\frac{300}{22}\right) = k \cdot t$

$\frac{\ln(300/22)}{k} = t$ $\quad \boxed{t \approx 75.616 \text{ years after } 1940}$

44. Determine whether f is an exponential function of x of the form $f(x) = ab^x$. If so, find the constant ratio.

x	-1	0	1	2	3
f(x)	0.667	1	1.5	2.25	3.375

$\frac{1}{.667} = 1.499$ $\quad \frac{1.5}{1} = 1.5$ $\quad \frac{2.25}{1.5} = 1.5$ $\quad \frac{3.375}{2.25} = 1.5$

YES... $b = 1.5$

$a = 1$ $\quad \boxed{y = 1(1.5)^x}$

45. The following table gives the population size of a flock of birds in one habitat over the last 57 years.

Years Since Data was First Collected	5	22	40	57
Population Size	18	22	85	185

a) Use exponential regression to find an exponential model for the data. Give the equation below:

$y = 11.264(1.049)^x$

b) Use logarithmic regression to find a logarithmic model for the data. Give the equation below:

$y = -97.765 + 56.390\ln(x)$

c) Using a scatterplot of the graph of the data along with the equations, which do you think is a better fit? EXPLAIN.

exponential

d) Using the equation you chose in part c, predict the population size 65 years after the data was first collected.

$y = 11.264(1.049)^{65} \approx \boxed{252.406}$

e) How long will it take for the population to reach 240?

$240 = 11.264(1.049)^x$

$\frac{240}{11.264} = 1.049^x$ $\nearrow$ $\frac{\log\left(\frac{240}{11.264}\right)}{\log(1.049)} \approx x$

$\log_{1.049}\left(\frac{240}{11.264}\right) = x$ $\quad \boxed{63.946 = x}$