Name $\boxed{\text{KEY}}$ Block ____ Date ____________

Chapter 3 Review

NON-CALCULATOR

1. On a separate piece of paper, thoroughly explain each of the following terms within the context of this chapter:

Binomial	Local maximum	Polynomial
Degree of a polynomial	Local minimum	Synthetic division
End behavior	Monomial	Trinomial
Leading coefficient	Multiplicity	Turning point

L.C. d # of terms

Rewrite each polynomial in standard form. Then, identify the leading coefficient, degree, and number of terms. Finally, name the polynomial.

2. $4x^2 - 3x^3 + 6x + 7$

$\boxed{-3x^3 + 4x^2 + 6x + 7}$

L.C = -3
deg 3
4 terms
CUBIC

3. $5x^3 - x^5 + 8x + 2x^4$

$\boxed{-x^5 + 2x^4 + 5x^3 + 8x}$

L.C = -1
deg = 5
4 terms
QUINTIC

3. $1 - 11x + 9x^2$

$\boxed{9x^2 - 11x + 1}$

L.C. = 9
deg = 2
3 terms
Quadratic

4. $-6x^2 + x^4$

$\boxed{x^4 - 6x^2}$

L.C. = 1
deg = 4
2 terms
QUARTIC

Add or Subtract. Write your answer in standard form.

5. $(8x^3 - 4x^3 + 6x + 7) - (1 - 5x^2 + x)$

$8x^3 - 4x^3 + 6x + 7 - 1 + 5x^2 - x = \boxed{4x^3 + 5x^2 + 5x + 6}$

6. $(6x^2 + 7x - 2) + (1 - 5x^3 + 3x)$ $= \boxed{-5x^3 + 6x^2 + 10x - 1}$

7. $(5x - 2x^2) - (4x^2 + 6x - 9)$

$5x - 2x^2 - 4x^2 - 6x + 9 = \boxed{-6x^2 - x + 9}$

Find each product.

8. $5x^2(3x - 2)$ $= \boxed{15x^3 - 10x^2}$

9. $-3t(2t^2 - 6t + 1)$

$\boxed{-6t^3 + 18t^2 - 3t}$

10. $ab^2(a^2 - a + ab)$ $= \boxed{a^3b^2 - a^2b^2 + a^2b^3}$

11. $(x - 2)(x^2 - 2x - 3)$

$x^3 - 2x^2 - 3x - 2x^2 + 4x + 6$

$\boxed{x^3 - 4x^2 + x + 6}$

12. $(2x + 5)(x^3 - x^2 + 1)$ $= 2x^4 - 2x^3 + 2x + 5x^3 - 5x^2 + 5$

$\boxed{2x^4 + 3x^3 - 5x^2 + 2x + 5}$

Divide using long division.

13. $(x^3 - 5x^2 + 2x - 7) \div (x + 2)$

$$
\begin{array}{r}
x^2 - 7x + 16 \\
x+2 \overline{\big)\, x^3 - 5x^2 + 2x - 7} \\
-(x^3 + 2x^2) \qquad\qquad \\
\hline
-7x^2 + 2x \qquad \\
-(-7x^2 - 14x) \qquad \\
\hline
16x - 7 \\
-(16x + 32) \\
\hline
-39
\end{array}
$$

14. $(8x^4 + 6x^2 - 2x + 4) \div (2x - 1)$

$$
\begin{array}{r}
4x^3 + 2x^2 + 4x + 1 \\
2x-1 \overline{\big)\, 8x^4 + 0x^3 + 6x^2 - 2x + 4} \\
-(8x^4 - 4x^3) \qquad\qquad\qquad \\
\hline
4x^3 + 6x^2 \qquad\qquad \\
-(4x^3 - 2x^2) \qquad\qquad \\
\hline
8x^2 - 2x \qquad \\
-(8x^2 - 4x) \qquad \\
\hline
2x + 4 \\
-(2x - 1) \\
\hline
\text{Remainder} = 5
\end{array}
$$

Divide using synthetic division

15. $(x^3 - 4x^2 + 3x + 2) \div (x - 3)$

$$
\begin{array}{r|rrrr}
3 & 1 & -4 & 3 & 2 \\
 & & 3 & -3 & 0 \\
\hline
 & 1 & -1 & 0 & \boxed{2}
\end{array}
\leftarrow \text{Remainder}
$$

16. $(x^3 + 2x - 1) \div (x - 2)$

$$
\begin{array}{r|rrrr}
2 & 1 & 0 & 2 & -1 \\
 & & 2 & 4 & 12 \\
\hline
 & 1 & 2 & 6 & \boxed{11}
\end{array}
\leftarrow \text{Remainder}
$$

17. A spool of ribbon has a length of $x^3 + x^2$ inches. Write an expression that represents the number of strips of ribbon with a length of $x - 1$ inches that can be cut from one spool.

$$
\begin{array}{r}
x^2 + 2x + 2 \\
x-1 \overline{\big)\, x^3 + x^2 + 0x + 0} \\
-(x^3 - x^2) \qquad\qquad \\
\hline
2x^2 + 0x \qquad \\
-(2x^2 - 2x) \qquad \\
\hline
2x + 0 \\
-(2x - 2) \\
\hline
2
\end{array}
$$

OR

$$
\begin{array}{r|rrrr}
1 & 1 & 1 & 0 & 0 \\
 & & 1 & 2 & 2 \\
\hline
 & 1 & 2 & 2 & \boxed{2}
\end{array}
$$

You can cut $x^2 + 2x + 2$ strips of ribbon w/ a little (2 in) left over.

18. Determine whether $(x - 1)$ is a factor of $P(x) = 4x^4 - 5x^2 + 3x - 2$

$$
\begin{array}{r|rrrr}
1 & 4 & -5 & 3 & -2 \\
 & & 4 & -1 & 2 \\
\hline
 & 4 & -1 & 2 & \boxed{0}
\end{array}
\leftarrow \text{Since Remainder} = 0, \ (x-1) \ \underline{\text{IS A FACTOR!}}
$$

Factor each expression below. GROUPING!

19. $x^3 - x^2 - 16x + 16$

	x	-1
x^2	x^3	$-x^2$
-16	$-16x$	16

$(x-1)(x^2-16)$

Difference of 2 perfect squares

$\boxed{(x-1)(x+4)(x-4)}$

20. $4x^3 - 8x^2 - x + 2$

	x	-2
$4x^2$	$4x^3$	$-8x^2$
-1	$-x$	$+2$

$(x-2)(4x^2-1)$

$\boxed{(x-2)(2x+1)(2x-1)}$

Write the simplest polynomial function with the given roots. (zeros) Leave your answer in factored form. → You DON'T HAVE TO MULTIPLY IT OUT!

21. $-3, 2, 4$

$\boxed{f(x) = (x+3)(x-2)(x-4)}$

22. $-\sqrt{2}, -1$

$+\sqrt{2}$ also

$\boxed{f(x) = (x+\sqrt{2})(x-\sqrt{2})(x+1)}$

23. $-3, i$ → $-i$ is too

$\boxed{f(x) = (x+3)(x-i)(x+i)}$

24. $1 + \sqrt{3}, 2i$

$1-\sqrt{3}$ & $-2i$ also

$\boxed{[x-(1+\sqrt{3})][x-(1-\sqrt{3})](x-2i)(x+2i)}$

Identify the leading coefficient, degree, and end behavior of the following polynomials.

L.C. deg

25. $-3x^6 + 9x^3 - 2x - 9$

$\swarrow \searrow$

L.C. = −3

deg = 6

$\boxed{\text{as } x \to \infty,\ y \to -\infty \quad \text{as } x \to -\infty,\ y \to -\infty}$

26. $7x^5 + x^4 - 2x^2 + 5$

$\swarrow \nearrow$

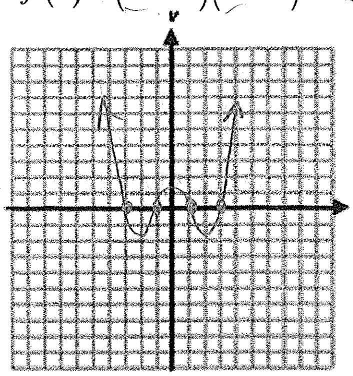
L.C. = 7

deg = 5

$\boxed{\text{as } x \to \infty,\ y \to \infty \quad \text{as } x \to -\infty,\ y \to -\infty}$

For 27 & 28, a) factor (or finish factoring) each to find the zeros, b) identify the end behavior, and c) use the answers to parts a and b to sketch the function.

27. $f(x) = (x^2 - 9)(x^2 - 1) = (x+3)(x-3)(x+1)(x-1)$



a) zeros: x = −3
x = 3
x = −1
x = 1

b) end behavior: $x^2 \cdot x^2 = x^4$ $\nwarrow \nearrow$

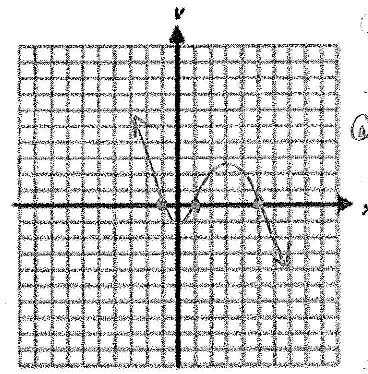
as $x \to \infty$, $f(x) \to \infty$

as $x \to -\infty$, $f(x) \to \infty$

28. $f(x) = -x^3 + 5x^2 + x - 5$

FACTOR BY GROUPING

	$-x$	5
x^2	$-x^3$	$5x^2$
-1	x	-5



ⓐ $(x^2 - 1)(-x + 5)$

$(x+1)(x-1)(-x+5)$

zeros: x = −1
x = 1
x = 5

ⓑ end behavior: $-x^3$

$\nwarrow \searrow$

Use the description to write each polynomial function in transformation form.

29. $f(x) = x^4$ is stretched vertically by a factor of 2, and is translated 9 units up to create $g(x)$.

$g(x) = 2x^4 + 9$

30. $f(x) = x^3$ is translated 2 units down and is reflected across the x-axis to create $g(x)$.

$g(x) = -x^3 - 2$

31. Give the parent function for $y = 3(x-2)^5 + 5$. Then, describe the transformations of the parent function.

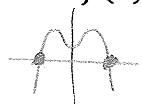
↳ $y = x^5$

↳ $(x+2,\ 3y+5)$ ...or... Right 2
up 5
vertically stretched by a factor of 2.

CALCULATOR

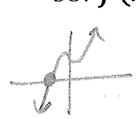
Graph each polynomial function on your calculator and then identify the number of real zeros.

32. $f(x) = -x^4 + 4x^2 + 1$



2 Real zeros

33. $f(x) = x^3 + 2x^2 + 1$



1 Real Zero

Identify all of the real roots of each equation.

34. $x^3 - 5x^2 + 8x - 4 = 0$

Graph... x = 2 looks a zero

$$\begin{array}{r|rrrr} 2 & 1 & -5 & 8 & -4 \\ & & 2 & -6 & 4 \\ \hline & 1 & -3 & 2 & \boxed{0} \end{array}$$

$x^2 - 3x + 2 = 0$

$x = \dfrac{3 \pm \sqrt{9 - 4(1)(2)}}{2(1)} = \dfrac{3 \pm \sqrt{1}}{2} = \dfrac{3 \pm 1}{2}$

$x = \dfrac{3+1}{2} = 2$ or $x = \dfrac{3-1}{2} = 1$

Roots
x = 2 (mult=2)
x = 1

35. $x^3 + 6x^2 + 9x + 2 = 0$

Graph... x = −2 looks like a zero

$$\begin{array}{r|rrrr} -2 & 1 & 6 & 9 & 2 \\ & & -2 & -8 & -2 \\ \hline & 1 & 4 & 1 & 0 \end{array}$$

$x^2 + 4x + 1 = 0$

$x = \dfrac{-4 \pm \sqrt{16 - 4(1)(1)}}{2(1)} = \dfrac{-4 \pm \sqrt{12}}{2} = \dfrac{-4 \pm 2\sqrt{3}}{2}$

$= -2 \pm \sqrt{3}$

Roots:
x = −2
$x = -2 + \sqrt{3}$
$x = -2 - \sqrt{3}$

36. $x^3 + 3x^2 + 3x + 1 = 0$

Graph... $x = -1$ Looks like a zero...

$$\begin{array}{r|rrrr} -1 & 1 & 3 & 3 & 1 \\ & & -1 & -2 & -1 \\ \hline & 1 & 2 & 1 & \boxed{0} \end{array}$$

$x^2 + 2x + 1 = 0$

$(x+1)(x+1) = 0$

$x = -1$ or $x = -1$

Roots: $x = -1$ (mult $= 3$)

37. $x^4 - 12x^2 + 27 = 0$

Graph... $x = -3$ & $x = 3$ look like zeros

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & -12 & 0 & 27 \\ & & -3 & 9 & 9 & -27 \\ \hline 3 & 1 & -3 & -3 & 9 & \boxed{0} \\ & & 3 & 0 & -9 & \\ \hline & 1 & 0 & -3 & \boxed{0} & \end{array}$$

$x^2 - 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

Roots: $x = -3$, $x = 3$, $x = \sqrt{3}$, $x = -\sqrt{3}$

Solve each equation by finding all roots.

38. $x^3 - x^2 + 4x - 4 = 0$

Graph... $x = 1$ Looks a zero

$$\begin{array}{r|rrrr} 1 & 1 & -1 & 4 & -4 \\ & & 1 & 0 & 4 \\ \hline & 1 & 0 & 4 & \boxed{0} \end{array}$$

$x^2 + 4 = 0$

$x^2 = -4$

$\sqrt{x^2} = \sqrt{-4}$

$x = \pm 2i$

Roots: $x = 1$, $x = 2i$, $x = -2i$

39. $x^4 - \frac{63}{4}x^2 - 4 = 0$

Graph... $x = 4$ looks like a zero

$$\begin{array}{r|rrrrr} 4 & 1 & 0 & -63/4 & 0 & -4 \\ & & 4 & 16 & 1 & 4 \\ \hline -4 & 1 & 4 & 1/4 & 1 & \boxed{0} \\ & & -4 & 0 & -1 & \\ \hline & 1 & 0 & 1/4 & 0 & \end{array}$$

also from graph

$x^2 + 1/4 = 0$

$x^2 = -1/4$

$\boxed{x = \pm\frac{1}{2}i}$

Roots: $x = 4$, $x = -4$, $x = \frac{1}{2}i$, $x = -\frac{1}{2}i$

40. $x^3 + 3x^2 - 5x - 15 = 0$

Graph... $x = -3$ looks like a zero

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -5 & -15 \\ & & -3 & 0 & 15 \\ \hline & 1 & 0 & -5 & \boxed{0} \end{array}$$

$x^2 - 5 = 0$

$x^2 = 5$

$x = \pm\sqrt{5}$

Roots: $x = -3$, $x = \sqrt{5}$, $x = -\sqrt{5}$

41. The following chart shows the attendance for a new movie theater over five days. Write a polynomial function for the data.

Day	1	2	3	4	5
Attendance	248	298	318	388	~~428~~ 588

50 20 70 200

-30 50 130

80 80

Since the 3rd differences are the same, the data is CUBIC

Run a CubicReg...

$\boxed{y = 13.\overline{3}x^3 - 95x^2 + 241.\overline{6}x + 88}$

42. The following chart shows the population of a city for five years. Write a polynomial function for the data.

Year	1	2	3	4	5
Population (thousands)	1891	2674	3376	4480	6469

783 702 1104 1989

-81 402 885

483 483

3RD Differences are the same,

CUBIC Reg...

$\boxed{y = 80.5x^3 - 523.5x^2 + 1790x + 544}$