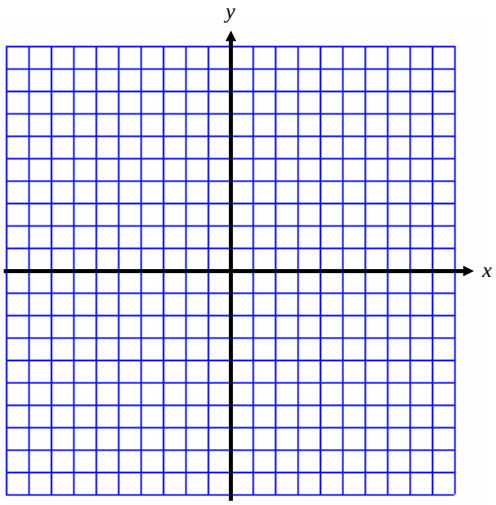
4.2 USING INVERSES OF EXPONENTIAL FUNCTIONS

The inverse of a linear function was a _____ function. The inverse of an exponential function is NOT exponential.

Example: Graph the exponential function $f(x) = 2^x$ by completing the following table.

x	$f(x)$
–3	
–2	
–1	
0	
1	
2	
3	



Example: Find the inverse numerically, and graph the inverse on the graph above.

Example: Complete the following:

$f^{-1}(8) =$ _____ $f^{-1}\left(\frac{1}{4}\right) =$ _____ $f^{-1}(1) =$ _____

Example: Rewrite each number inside the parenthesis above as a power of 2.

$f^{-1}\left(2^{\square}\right) =$ _____ $f^{-1}\left(2^{\square}\right) =$ _____ $f^{-1}\left(2^{\square}\right) =$ _____

Example: What problem do you encounter when you try to find the inverse of $f(x) = 2^x$ algebraically?

Logarithmic Functions

If $f(x) = 2^x$, then $f^{-1}(x) =$ _____, which is a logarithmic function with a base of _____.

Example: If $f(x) = 3^x$, then $f^{-1}(x) =$ _____ ?

Definition of a Logarithmic Function

You need to learn how to use this definition in <u>3 ways</u>.

1. Write a logarithmic equation as an exponential equation.
2. Write an exponential equation as a logarithmic equation.
3. Evaluate logarithmic expressions with or without your calculator

#1: Write a logarithmic equation as an exponential equation.

Example: Write each equation in exponential form.

a) $\log_3 81 = 4$ b) $\log_7 343 = 3$ c) $\log_{16} \frac{1}{4} = -\frac{1}{2}$ d) $\log_5 \frac{1}{25} = -2$

#2: Write an exponential equation as a logarithmic equation.

Example: Write each equation in logarithmic form.

a) $4^2 = 16$ b) $6^{-2} = \frac{1}{36}$ c) $25^{\frac{1}{2}} = 5$ d) $10000^{\frac{1}{4}} = 10$

#3: Evaluate logarithmic expressions with or without your calculator

Example: Evaluate each expression without using your calculator.

a) $\log_3 81 =$ b) $\log_2 32 =$ c) $\log_9 \frac{1}{729} =$ d) $\log_7 1 =$

e) $\log_{16} 8 =$ f) $\log_{64} 512 =$ g) $\log_{\frac{1}{2}} 8 =$ h) $\log_{27} \frac{1}{9} =$

There are two logarithms your calculator CAN do, and we use specific notation for each.

#1: A *common logarithm* has a base of 10, and $\log_{10} x =$ _____

#2: A *natural logarithm* has a base of *e*, and $\log_e x =$ _____

Example: Write each equation in exponential form.

a) $\log 0.0001 = -4$ b) $\ln 1 = 0$

Example: Write each equation in logarithmic form.

a) $10^7 = 10000000$ b) $e^2 = 7.389$

Example: Evaluate each logarithmic expression.

a) $\log 1000 =$ b) $\log \frac{1}{100} =$ c) $\ln e^4$ d) $\ln e^{-\frac{4}{3}}$

Example: Using your calculator, evaluate the following expressions.

a) $\log 12$ b) $\ln 5$ c) $\log\left(\frac{4}{9}\right)$

To use your calculator effectively, you must be able to rewrite logarithms with bases other than 10 or *e*. To do this we have the *change of base formula*.

Change of Base Formula

$$\log_b a =$$

Example: Evaluate each logarithmic expression using your calculator.

a) $\log_3 15$ b) $\log_7 13$

Example: Consider the equation $5 = 2^x$.

a) Solve the equation graphically. b) Solve the equation algebraically.

Example: Solve $10 = 5(1.06)^x$ for *x*.

Example: Solve $7^{3x} = 20$ for *x*.

Example: If $f(x) = 2(3)^x$, then find $f^{-1}(x)$.

When you find inverses of application problems, you don't switch the variables, you simply solve for the other variable.

Example: The population of Nepal (in millions) can be modeled by the function $P = 19.1e^{0.025t}$, where *t* is the number of years since 1990.

a) What year will the population of Nepal reach 30 million?

b) What year will the population of Nepal reach 15 million?

c) Find the inverse of the given function. What information can you obtain with the inverse?