3.5 FITTING EXPONENTIAL FUNCTIONS TO DATA

Skills you are going to need today

Example: Simplify each of the following exponential expressions. Write all answers with positive exponents only.

a) \( \left( \frac{1}{8} \right)^{\frac{1}{2}} \)  

b) \( \left( \frac{27}{64} \right)^{\frac{1}{3}} \)  

c) \( \left( \frac{1}{32} \right)^{\frac{1}{6}} \)  

d) \( \frac{3x^5}{x^2} \)

e) \( \frac{2b^{\frac{3}{2}}}{b^{\frac{1}{2}}} \)

f) \( \left( \frac{3}{4a^4} \right)^a \)  

g) \( \frac{24}{b^{\frac{1}{3}}b^7} \)

Example: Solve each of the following equations for \( x \).

a) \( x^2 = 25 \)  

b) \( x^3 = 8 \)  

c) \( 8x^3 = 512 \)

d) \( 27 = 125x^3 \)  

e) \( \frac{1}{x^4} = 12x^3 \)

Writing Exponential Functions

You goal today is to learn how to write an exponential function \( y = ab^x \) that includes certain points. First, we will do this without a calculator, and then we will use a calculator (ExpReg). In both cases, you are trying solve for \( \boxed{} \).

Example: Write an exponential function \( y = ab^x \) for a graph that includes the points (2, 2) and (5, 16).

a) STEP 1: Write two equations by plugging both points into the equation \( y = ab^x \).

b) STEP 2: Solve for \( a \) in the first equation.

c) STEP 3: Substitute this expression for \( a \) into the second equation, and solve for \( b \).

d) STEP 4: Using your expression for \( a \) and your solution for \( b \), solve for \( x \).

e) STEP 5: Write the exponential equation.
3.5 Fitting Exponential Functions to Data

Example: Write the exponential equation for the graph that includes the following points.

a) (1, 15) and (2, 45).

b) (3, 8) and (6, 512)

c) (–1, 12) and (2, \( \frac{1}{16} \))

d) Challenge: (–1, \( \frac{25}{3} \)) and (2, \( \frac{9}{7} \))
3.5 Fitting Exponential Functions to Data

Now, let’s use your calculator. Fitting an Exponential Function to Data using your calculator is exactly the same as using your calculator to find the linear regression line. Except we call it an Exponential Regression or ______________ on your calculator.

**Example:** The table at the right shows the number of degrees above room temperature for a cup of coffee after \(x\) minutes of cooling. Use your calculator to find the exponential function that models this data.

<table>
<thead>
<tr>
<th>Time</th>
<th>°F above room temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>135</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>74</td>
</tr>
<tr>
<td>15</td>
<td>55</td>
</tr>
<tr>
<td>20</td>
<td>41</td>
</tr>
<tr>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td>30</td>
<td>22</td>
</tr>
<tr>
<td>35</td>
<td>17</td>
</tr>
<tr>
<td>40</td>
<td>12</td>
</tr>
</tbody>
</table>

a) Enter the time in List1 and the temperature in List 2.
(\([\text{STAT}], 1:\text{EDIT}\))

b) View your data on the screen
(\([\text{STAT PLOT}]\) to turn PlotsOn, then \([\text{ZOOM}], 9:\text{ZoomStat}\) to view data points)

c) Find exponential equation. Write it down.
(\([\text{STAT}], \[x], 0:\text{ExpReg})\))

d) Graph the function on the screen
(\([y=], \text{VARS}, 5:\text{Statistics}, [x], [y], 1:\text{RegEQ})\))

If you only need to find the equation, then you only need to do steps \(a\) and \(c\) above.

e) In the context of this problem, what does \(a\) stand for?

f) In the context of this problem, what does \(b\) stand for?

g) Find the line of best fit for this example. Which is a better fit? Explain.

**Example:** Use your calculator to find the exponential equation that models the following sets of data.

\[
\begin{array}{ccccccc}
  x & -3 & -2 & -1 & 0 & 1 & 2 \\
  y & 50 & 25 & 12.5 & 6.25 & 3.13 & 1.56 \\
\end{array}
\]

\[
\begin{array}{ccccccc}
  x & 0 & 1 & 2 & 3 & 4 & 5 \\
  y & 2 & 2.4 & 2.88 & 3.46 & 4.15 & 5 \\
\end{array}
\]
Example: The number of United States citizens $y$ (in millions) who traveled to foreign countries in the years 1988 through 1996 are shown in the table below, where $t = 8$ represents the year 1988.

<table>
<thead>
<tr>
<th>$t$</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>40.7</td>
<td>41.1</td>
<td>44.6</td>
<td>41.6</td>
<td>43.9</td>
<td>44.4</td>
<td>46.5</td>
<td>50.8</td>
<td>52.3</td>
</tr>
</tbody>
</table>

a) Use the regression capabilities of your graphing calculator to find an exponential model that fits the data.

b) In the context of this problem, what does the $a$ value represent?

c) In the context of this problem, what does the $b$ value represent?

d) According to the model, is the number of travelers increasing or decreasing? At what rate?

e) Using your model, how many travelers were there in 1980? 1974? 2000?

f) According to this model, when will there be 80 million travelers?