3.4 THE NUMBER e

One of the applications of exponential functions is compound interest. We are going to learn about two formulas that involve compounding interest. Your goal today is to learn when to apply each one. The two formulas are

Example: Suppose you invest $1000 in an account that earns you 4% interest at the end of every year.

a) How much interest do you get at the end of the first year?

b) How much money do you have in your account at the end of the first year?

c) How much money do you have in your account at the end of the second year?

d) If you haven't already, can you write an exponential equation that models this for t years?

e) How much money do you have in your account at the end of 6 years?

Example: Suppose that you invest the same $1000 in an account that earn you 4% interest, but they give you interest at the end of every month instead of waiting till the end of the year. (This is called compounding your interest monthly).

a) Since your interest rate is 4% per year, what is your interest rate for each month?

b) What is the growth factor?

c) Write an exponential equation that models this situation … explain how the exponent is defined.

d) How much money would you have in your account after 1 month?

e) How much money would you have in your account after 1 year?

f) How much money would you have in your account after 6 years?

Example: Go back and look at the first equation at the top of the page … does it make more sense?

Example: Find the amount of money in your account if you invested $2500 at 5% interest compounded weekly for 3 years.

Let's take a detour for a minute in dealing with compound interest.

Are you familiar with the irrational number π? We are now going to introduce another irrational number, the number *e*.

Example: Consider the expression $\left(1+\frac{1}{n}\right)^n$. Use your calculator to complete the following table for different values of *n*.

Round each value to 6 decimal places.

n	$\left(1+\frac{1}{n}\right)^n$
1	
10	
100	
1,000	
10,000	
100,000	
1,000,000	
10,000,000	

The expression $n \to \infty$ means ____________________________. We define *e* using the expression $\left(1+\frac{1}{n}\right)^n$.

As $n \to \infty$, $\left(1+\frac{1}{n}\right)^n \to e \approx$ ________________.

Do you notice how similar $\left(1+\frac{1}{n}\right)^n$ looks to the equation $\left(1+\frac{r}{n}\right)^{nt}$?

Example: In the second expression, *n* stands for ____, so how often are you compounding the interest if $n \to \infty$?

We could show or investigate how they are similar, but how about if I just tell you? It turns out, if $n \to \infty$ (or we are compounding the interest continuously), that $\left(1+\frac{r}{n}\right)^{nt} \to e^{rt}$. All this means that if you are compounding interest continuously, use the formula _________________.

Example: Suppose you invest your $1000 into an account that earns 4% interest compounded continuously.

a) How much money is in your account after 6 years?

Example: Find the value of an investment of $2000 after four years if the interest rate is 4.5% and interest is compounded as specified below.

a) annually

b) weekly

c) continuously