3.1 EXPONENTIAL GROWTH AND DECAY

We have spent the entire time in chapter 2 dealing with linear functions only. In chapter 3 we are going to deal with a different type of function called *exponential functions*. Your first job in this chapter is to understand the difference between exponential growth and decay and to write equations of exponential growth/decay functions.

Consider the following situation: You have been offered a wonderful job doing something you've always wanted to do. The only thing keeping you from taking the job is your concern over the amount and method of payment. The terms of the job are somewhat interesting. The job requires 30 days to complete, and your potential future boss has offered to pay you $0.01 for your first full day of work, $0.02 for your second day of work, $0.04 for your third day of work, and you are promised that your pay will continue to double every day until you finish the 30 days of work.

Example: First impressions ... do you want this job?

Example: Using a calculator, complete the following chart to keep track of the amount of money you could make. For the purposes of this example, we are going to let x = the number of days after the first day.

Day #	Pay for that day	Total Pay to Date
0	$0.01	$0.01
1	$0.02	$0.03
2	$0.04	$0.07
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		

Day #	Pay for that day	Total Pay to Date
15		
16		
17		
18		
19		
20		
21		
22		
23		
24		
25		
26		
27		
28		
29		

Example: Write an equation that shows the amount you make each day as a function of the number of days.

Example: Write an equation that shows the total pay to date as a function of the number of days.

Example: Using your equation above, if you were to work for 45 days, what would your pay be for the 45^{th} day?

Example: Enter the data above into your graphing calculator.

Example: View your data on your calculator and draw what you see on your screen. Describe your window.

Example: Run an ExpReg on your data. What is the equation your calculator gives you? Graph this equation on your calculator and draw the graph on your sketch from the last example.

Consider this situation: You decided to purchase a car for $28,500. According to a consumer magazine, this particular car loses 12% of its value every year.

Example: Using a calculator, complete the chart below to keep track of how much your car will be worth each year.

# of Years	Value of Car
0	$28,500
1	$25,080
2	
3	
4	
5	
6	

Example: Write an equation that shows the value of your car as a function of the number of years.

Example: Using your equation above, if you were to keep your car for 10 years, what would it be worth?

Example: Enter the data above into your graphing calculator.

Example: View your data on your calculator and draw what you see on your screen. Describe your window.

Example: Run an ExpReg on your data. What is the equation your calculator gives you? Graph this equation on your calculator and draw the graph on your sketch from the last example.

The previous two situations are examples of exponential growth and decay.

The equation for <u>exponential growth</u> is

where a = _____, b = _____, x = _____, and y = _____.

The equation for <u>exponential decay</u> is

where a = _____, b = _____, x = _____, and y = _____.

Example: Identify the following equations as exponential growth or decay. What was the initial amount? What is the growth or decay rate?

a) $y = 5(1.47)^x$ b) $y = 3.2(.72)^x$

Example: A certain medication is eliminated from the bloodstream at a rate of about 20% per hour. The original dosage of medication is 40 mg.

a) Write an exponential function that models the amount of medication left in the bloodstream after *x* hours.

b) According to your equation, how much of the medication is still in the bloodstream after 6 hours?

Example: A certain intersection in the city had 45 reported traffic accidents in 1997. The number of accidents have been increasing by 4% each year.

a) Write an exponential function that models the number of reported accidents *x* years after 1997.

b) According to your equation, how many reported accidents will there be in the year 2010?

Exponent Rules … An Introduction

First things, first. Understanding what value you should apply the exponent to is extremely important.

Example: Identify the base in each of the following expressions.

a) -4^2 b) $(-4)^2$ c) $(-3)^3$ d) -3^3

e) $2x+1^2$ f) $(2x+1)^2$

Rules to Remember:

$a^m \cdot a^n =$

$\frac{a^m}{a^n} =$

$\left(a^m\right)^n =$

Example: Using the rules above, simplify each of the following expressions.

a) $2^3 \cdot 2^4 =$ b) $\frac{y^8}{y^3} =$ c) $\left(q^3\right)^4$

Example: Simplify the following expressions.

a) $\left(-4a^2b^3\right)\left(3ab^6\right)^2$ b) $\frac{16x^3\left(y^6z^2\right)^4}{\left(2xy^3z\right)^2}$